Øving

Løs likningen

11.02 2022

Opg $\log(x+2) = 2\log(x)$ $\qquad x > 0$

$$\Rightarrow \log_{10}(x+2) = \log_{10}(x^2)$$

$$x+2 = x^2$$

$$x^2 - x - 2 = 0$$

$$\left(x-\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2 = 0$$

$$\left(x-\frac{1}{2}\right)^2 = 2 + \frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$$

$$x = \frac{1}{2} \pm \frac{3}{2}, \qquad x = 2 \text{ og } x = -1$$

Skal ha $x > 0$ så det er bare én løsning

$$\underline{x = 2}$$

Nullpunkt til $x e^{(x-1)}$.

$$x \cdot e^{(x-1)} = 0 \iff x = 0 \text{ fordi}$$

$e^x > 0$ for alle x

$$x e^x - 1 = 0 \iff x e^x = 1.$$

Numerisk: $x \sim \underline{0.56714}$

opg. fra cosinus

$$\ln\left(\sqrt{e}\,\sqrt[3]{e}\,\sqrt[6]{e}\right) = \ln\left(e^{1/2} e^{1/3} e^{1/6}\right)$$

$$= \ln\left(e^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}\right) = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

8.143d)

$$= \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{3+2+1}{6} = \underline{\underline{1}}$$

8.153 b) $e^{x} - 7 + 10e^{-x} = 0$, $e^{-x} = \frac{1}{e^{x}}$

$u - 7 + \frac{10}{u} = 0$ $u = e^{x}$

$x = \ln u$.

$$\frac{u^2 - 7u + 10}{u} = 0$$

$\Leftrightarrow$ $u^2 - 7u + 10 = 0$

$(u-5)(u-2) = 0$

Så $u = 2$ og $u = 5$.

Løsningene er $\underline{x = \ln 2}$ og $\underline{\ln 5}$

opg 8.21 b) $3 \cdot 2^x = 5$

$$2^x = \frac{5}{3}$$

$$\log(2^x) = \log\left(\frac{5}{3}\right) = \log\left(5 \cdot \frac{1}{3}\right)$$

$$x \log 2 = \log 5 - \log 3$$

$$x = \frac{\log(5) - \log(3)}{\log(2)}$$

Alternativt:

$$\log(3 \cdot 2^x) = \log 5$$

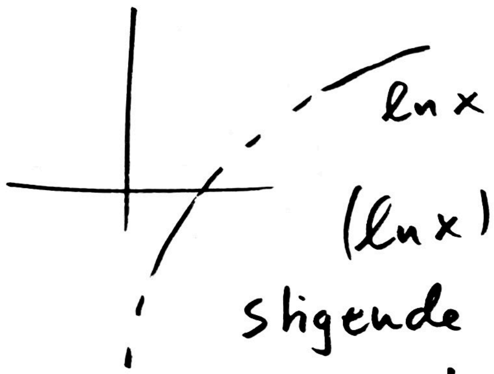
$$\log(3) + \log 2^x = \log 5$$

$$x \log 2 = \log 5 - \log 3$$

$$x = \frac{\log 5 - \log 3}{\log 2}$$

cosinus: 8.227 b) $e^x - 4e^{-x} > 0$

La $u = e^x > 0$



$(\ln x)' = \frac{1}{x} > 0$

stigende

$a < b \Leftrightarrow \ln a < \ln b$

og $a < b \Leftrightarrow e^a < e^b$

$u - \frac{4}{u} > 0$

$\frac{u^2-4}{u} > 0$

Fortegnskjema

		-2		0		2	
$(u-2)$	- - -	- - -	- - -	- - -	- - -	0	———
$(u+2)$	- - -	0	———	———	———	———	———
$1/u$	- - -	- - -	- - -	×	———	———	———
$\frac{u^2-4}{u}$	- - -	0	———	×	- - -	0	———

for $u > 0$ så er $\frac{u^2-4}{u} > 0$

for $u > 2$, $e^x > 2$

$\Leftrightarrow \underline{x > \ln 2}$

Deriver :

1) $\frac{(x+2)^3}{e^{2x}} = (x+2)^3 \cdot e^{-2x}$ så $\left(\frac{(x+2)^3}{e^{2x}}\right)' = \underbrace{\left((x+2)^3\right)'}_{3(x+2)^2(x+2)'} e^{-2x}$

$+ (x+2)^3 \cdot \underbrace{\left(e^{-2x}\right)'}_{e^{-2x}(-2x)'} = 3(x+2)^2 e^{-2x} - 2(x+2)^3 e^{-2x}$

$= (x+2)^2 e^{-2x}(3-2(x+2))$

$= -(2x+1)(x+2)^2 e^{-2x}$

2) $g(x) = \operatorname{Log}\left(e^{x^2+3x}(4-x)^5\right)$

$= \frac{\ln}{\ln 10}\left(e^{x^2+3x}(4-x)^5\right) = \frac{1}{\ln 10}\left[\ln e^{x^2+3x} + \ln(4-x)^5\right]$

$= \frac{1}{\ln 10}\left[x^2+3x + 5\ln(4-x)\right]$

$g'(x) = \frac{1}{\ln 10}\left(2x+3 + 5\frac{1}{4-x}\underbrace{(4-x)'}_{-1}\right)$

$= \frac{1}{\ln 10}\left(2x+3 - \frac{5}{4-x}\right)$