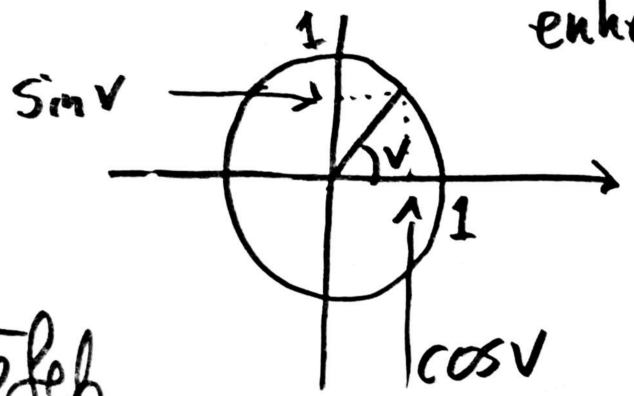
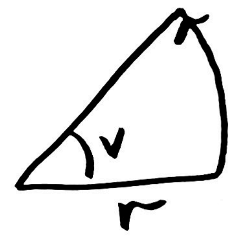
Kap 11 Trigonometriske funksjoner

positiv retning



enhetssirkel
(radius 1)



buelengde = b

15 feb.
2022

vinkel i radianer $\frac{b}{r} = \frac{\text{buelengde}}{\text{radius}}$

$$360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

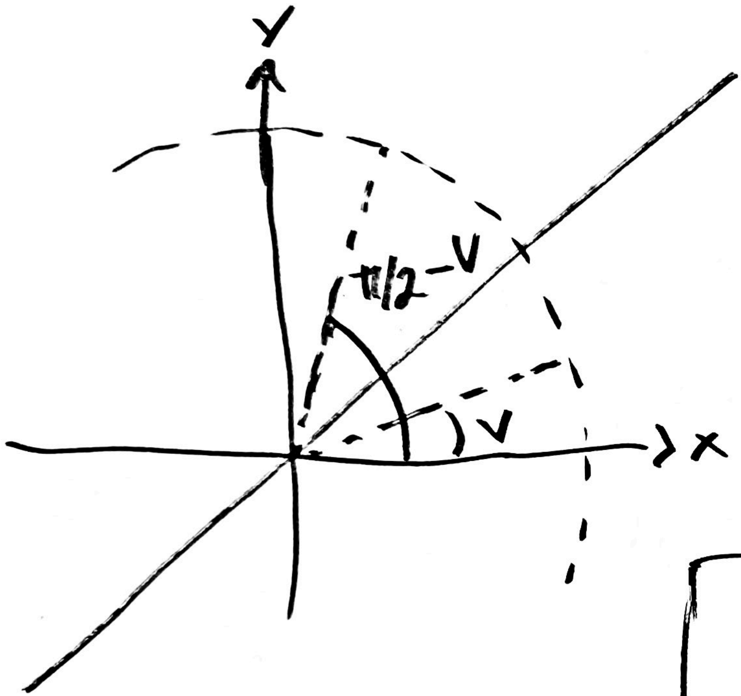
Radianer : naturlig vinkelmål

$\sin x$, $\cos x$ definert for alle $x \in \mathbb{R}$

De er <u>periodiske</u> med periode 2π :

$$\boxed{\begin{aligned} \sin(x+2\pi) &= \sin x \\ \cos(x+2\pi) &= \cos(x) \end{aligned}}$$

Pytagoras : $\boxed{\cos^2 x + \sin^2 x = 1}$ for alle x



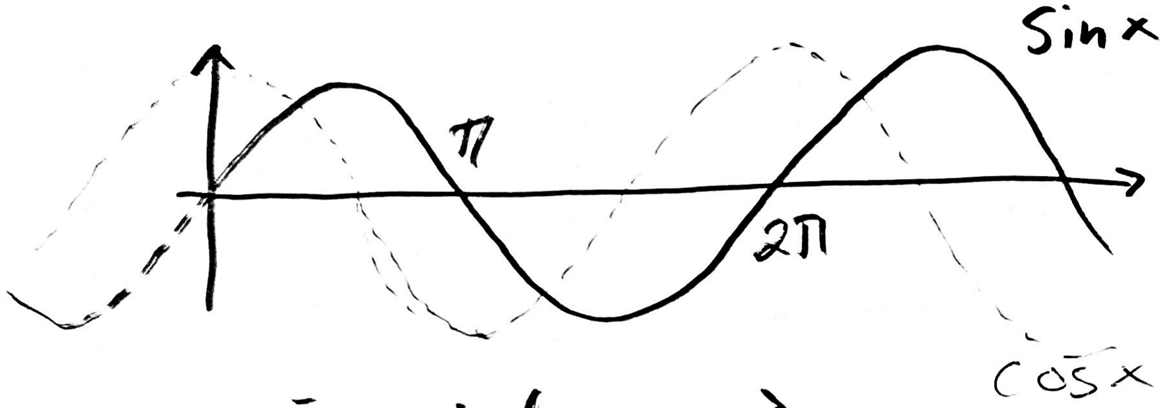
Reflekteren om linjen $x=y$

x og y-aksene bytter plass.

En vinkel v sendes til vinkel $\frac{\pi}{2} - v$.

Dette gir:

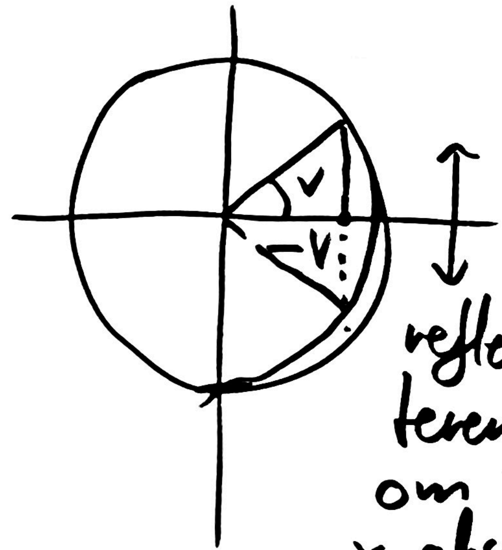
$$\boxed{\begin{aligned} \sin v &= \cos\left(\frac{\pi}{2} - v\right) \\ \cos v &= \sin\left(\frac{\pi}{2} - v\right) \end{aligned}}$$



← én periode →

$\sin(-v) = -\sin(v)$ odde funksjon

$\cos(-v) = \cos(v)$ jevn funksjon



reflekterer om x-aksen

$\underline{\sin\left(v+\frac{\pi}{2}\right)} = \cos(-v) = \underline{\cos v}$

Grafen til cos v ~~er~~ lik grafen til sin v forskyvd med $-\frac{\pi}{2}$ ($\frac{\pi}{2}$ mot venstre)

Addisjonsformlene for sin og cosin.

$$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cdot\cos(u)$$

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$

tester litt. $\sin(u+0) = \sin(u)\underbrace{\cos(0)}_{1} + \underbrace{\sin(0)}_{0}\cos(u)$

$$= \sin(u) \checkmark$$

$$\sin\left(\frac{\pi}{2} - v\right) = \sin\left(\frac{\pi}{2} + (-v)\right)$$

$$= \underbrace{\sin\frac{\pi}{2}}_{1}\cos(-v) + \sin(-v)\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}$$

$$= \cos(-v) = \cos(v) \ldots$$

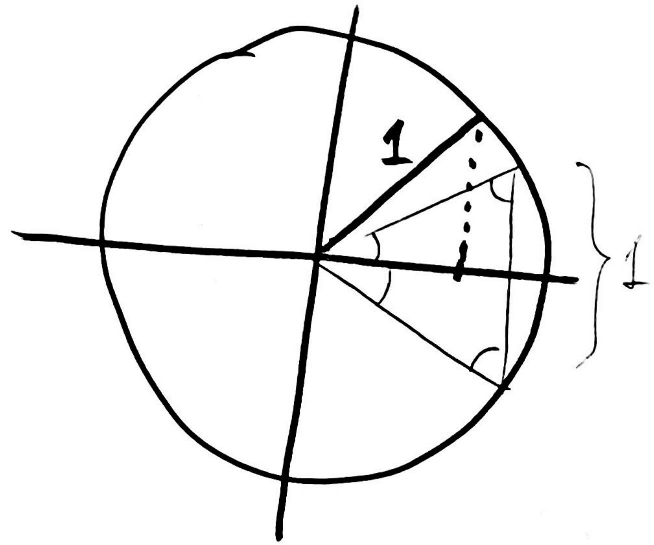
$\sin(0) = 0$ $\qquad \sin\left(\frac{\pi}{6}\right) = 1/2$ $\qquad \sin(\pi/4) = 1/\sqrt{2} \sim 0.707$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \sim 0.866$ $\qquad \left[\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}\right]$

$$\sin(75^\circ) = \sin(30^\circ + 45^\circ)$$

$$= \underbrace{\cos(30^\circ)}_{\frac{\sqrt{3}}{2}} \underbrace{\sin(45^\circ)}_{\frac{1}{\sqrt{2}}} + \underbrace{\cos(45^\circ)}_{\frac{1}{\sqrt{2}}} \underbrace{\sin(30^\circ)}_{1/2}$$

$$\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}} \sim 0.966$$



11.2 Dobling av vinkel

$$\boxed{\begin{aligned} \sin(2v) &= \sin(v+v) = 2\sin(v)\cos(v) \\ \cos(2v) &= \cos(v+v) = \cos^2(v) - \sin^2(v) \end{aligned}}$$

Legger til $1 = \cos^2 v + \sin^2 v$ gir

$$\cos^2(v) = \frac{1}{2}(1 + \cos(2v))$$

$$\sin^2(v) = \frac{1}{2}(1 - \cos(2v))$$

Opg: Finn eksakt verdi til $\sin(22{,}5°)$.

(Hint: $2 \cdot 22.5° = 45°$) La $v = 22.5°$

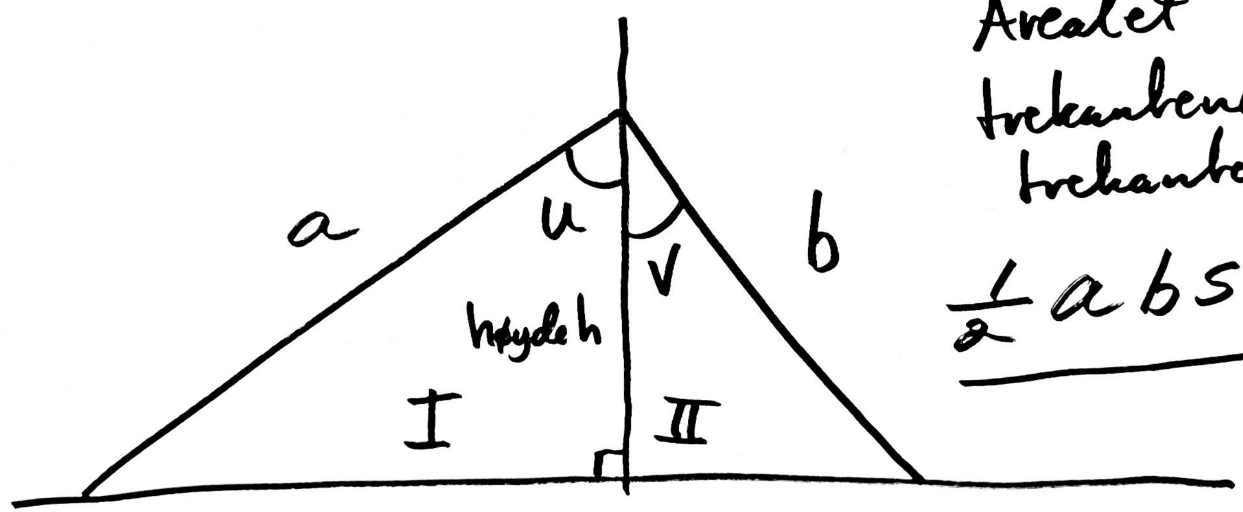
$$\sin^2(22.5°) = \frac{1}{2}(1 - \underbrace{\cos(45°)}_{1/\sqrt{2}}) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

$\sin(22.5°) > 0$ så

$$\sin(22.5°) = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2}\,\sqrt[4]{2}}$$

$$\sim 0.38268\ldots$$

Bevis for addisjonsformelen



Arealet til de to trekantene (den store trekanten)

$$\frac{1}{2} a b \sin(u+v)$$

Areal trekan I : $\dfrac{a \cdot \sin u \cdot h}{2}$

Areal trekant II : $\frac{b \sin v \cdot h}{2}$

høyden $h = a \cdot \cos u = b \cos v$

sette inn i uttrykket for A_{II}

sette inn i uttrykket for A_I

$$A_I + A_{II} = \frac{1}{2} ab \sin(u+v)$$

$$\frac{a \sin u \, (b \cos v)}{2} + \frac{b \sin v \cdot a \cos u}{2}$$

$$\frac{ab}{2}(\sin u \cos v + \sin v \cos u) = \frac{ab}{2} \sin(u+v)$$

Så $\sin(u+v) = \sin u \cos v + \sin v \cos u$.

$0 \leq u, v \leq \frac{\pi}{2}$.

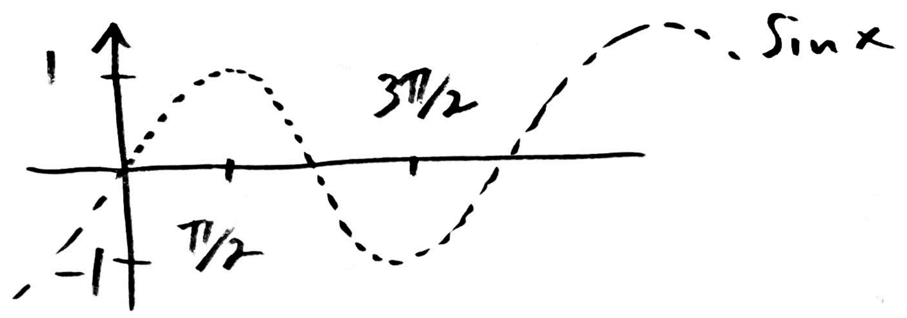
utvides til alle u, v.

Addisjonsformel for cos følger fra den for sin

$$\cos(u+v) = \sin\left(\left(\frac{\pi}{2} - u\right) - v\right)$$

$$= \underbrace{\sin\left(\frac{\pi}{2} - u\right)}_{\cos(u)} \underbrace{\cos(-v)}_{\cos(v)} + \underbrace{\sin(-v)}_{-\sin(v)} \underbrace{\cos\left(\frac{\pi}{2} - u\right)}_{\sin(u)}$$

$$= \underline{\cos(u)\cos(v) - \sin(u)\cos(v)}.$$



konkav: $\frown$ $\smile$

toppunkt i $x = \frac{\pi}{2} + 2\pi \cdot n$ $\quad n \in \mathbb{Z}$

bunnpunkt : $x = \frac{3\pi}{2} + 2\pi \cdot n$

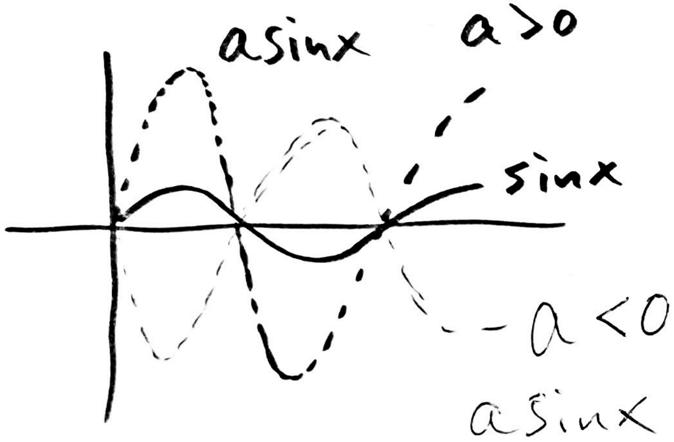
opg:

Hvor har $\sin(3x - 1)$ toppunkt?

Det er for x s.a $3x - 1 = \frac{\pi}{2} + 2\pi \cdot n$

$$\underline{x = \frac{1}{3}\left(\frac{\pi}{2} + 1\right) + \frac{2\pi}{3} \cdot n}$$

$a \sin x$



topp- og bunn-punkt

Toppunkt i $\qquad n \in \mathbb{Z}$

$\left(\frac{\pi}{2} + 2\pi \cdot n,\ a\right) \qquad a > 0$

$\left(\frac{3\pi}{2} + 2\pi \cdot n,\ -a\right) \qquad a < 0$

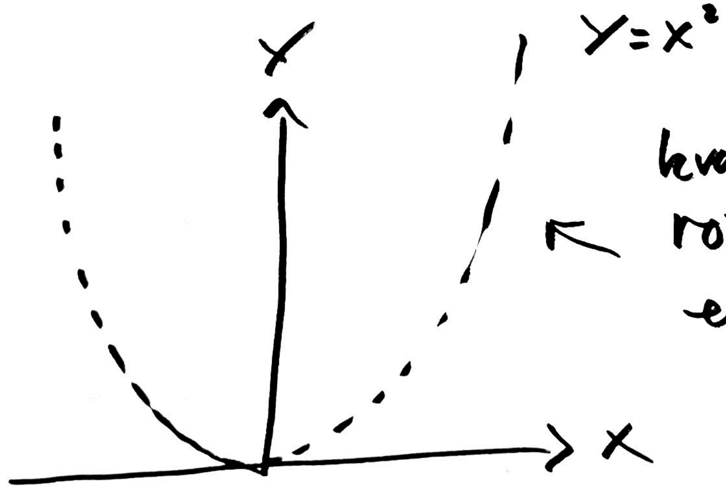
$(x, 0)$ for alle $x \qquad a = 0$

$\sqrt{4} = 2$

$x^2 = 4$ løsninger $x = -2$, $\underline{x = 2}$

$\sqrt{x}$ funksjon (ēn verdi)

løsningen til likningen $y^2 = x$ hva $y \geq 0$.



kvadrat-
rotfunksjonen
er inversfunksjonen til
$f(x) = x^2$
med $D_f = [0, \infty \rangle$.

11.4 Sinusfunksjoner.

$$a \sin(kx + c) + d$$

amplitude $|a|$

$y = d$ likevektslinje

$2\pi = k \cdot P$ så perioden $\underline{P = \frac{2\pi}{k}}$

Faseforskyvning. $kx + c = k\left(x + \frac{c}{k}\right)$

Grafen forskyves med $-\frac{c}{k}$.