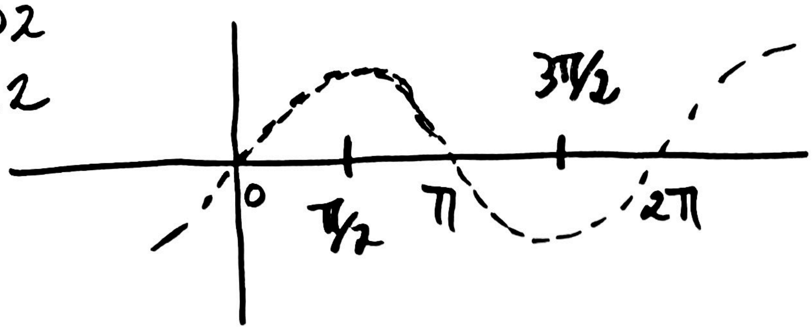
16.02
2022



sinx

periode 2π

$$y(x) = a\sin(kx + c) + d$$

$|a|$ amplitude

$y = d$ likevektslinjen (horisontal)

Periode $P = \frac{2\pi}{k}$

Faseforskyvning $\frac{-c}{k}$

Eksempel $2\sin(x+\pi) - 3$

amplitude 2

likevektslinje $y = -3$

periode 2π

Faseforskyvning

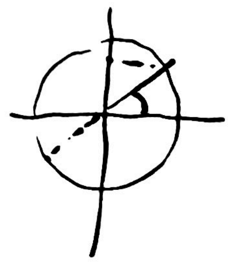
$-\pi \quad (+2\pi \cdot n)$

* $f(x) = -3(\sin(2\pi x - \pi) - 1)$

amplitude $3 = |-3|$

likevektslinjen : $y = 3$

Perioden : $\frac{2\pi}{k} = \frac{2\pi}{2\pi} = 1$



Faseforskyvning : $-\sin(2\pi x - \pi)$

$= \sin(2\pi x)$

Faseforskyvningen er lik 0.

Finn toppunkt til

$$f(x) = -3\sin(2x+1) + 2.$$

er størst når $\sin(2x+1)$ er minst.

$$\sin(2x+1) = -1$$

Da er $2x+1 = \frac{3\pi}{2} + 2\pi \cdot n$
$n \in \mathbb{Z}$

Funksjonsverdien er da lik
$$-3(-1)+2 = 5.$$

$$2x = \frac{3\pi}{2} - 1 + 2\pi \cdot n$$

$$x = \frac{3\pi}{4} - \frac{1}{2} + \pi \cdot n$$

$$x \sim \underline{1,8561\ldots + \pi \cdot n}$$

Toppunktene er $\underline{\left(\frac{3\pi}{4} - \frac{1}{2} + \pi \cdot n,\ 5\right)}$
for $n \in \mathbb{Z}$

Finn bunnpunkt til

$$g(x) = -2\sin(x) + 4$$

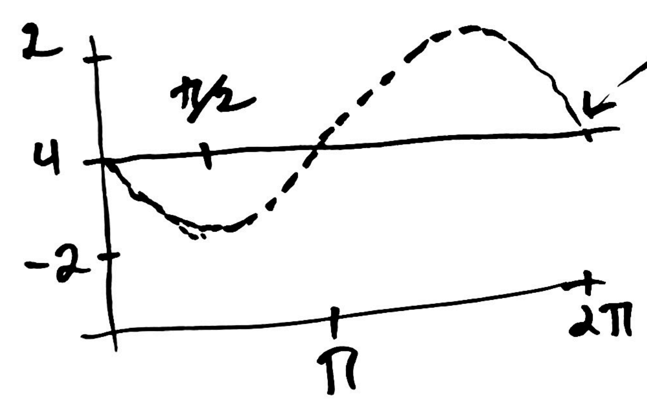
$$x \in [0, 2\pi]$$

Globale bunnpunkt hvor $\sin x = 1$

$$x = \frac{\pi}{2}.$$

$$g\left(\frac{\pi}{2}\right) = -2(+1) + 4 = +2.$$

$$\underline{\left(\frac{\pi}{2}, +2\right)}$$

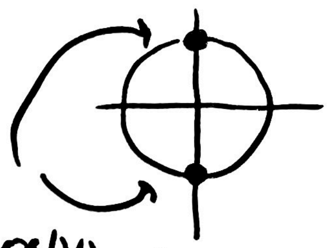


lokalt bunnpunkt

$$\underline{(2\pi, 4)}$$

11.6 Tangens

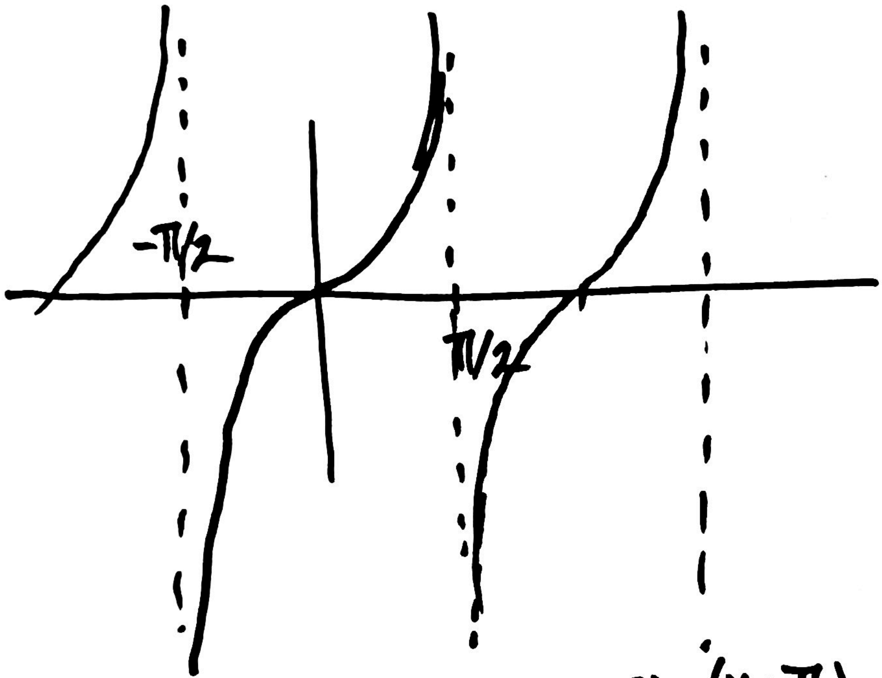
$$\tan(V) = \frac{\sin(V)}{\cos(V)}$$



cos(V) = 0

for $\cos V \neq 0$

$V \neq \frac{\pi}{2}, \frac{3\pi}{2}$ opp til hele omløp.



$$\tan(x+\pi) = \frac{\sin(x+\pi)}{\cos(x+\pi)} = \frac{-\sin(x)}{-\cos(x)} = \tan(x)$$

tan(x) periodisk med periode π

ingen topp- eller bunnpunkt.

Vertikale asymptoter $x = \frac{\pi}{2} + \pi \cdot n$

cotangens $\cot(x) = \frac{\cos x}{\sin x}$

Definert for $\sin x \neq 0$ $n \in \mathbb{Z}$

d.v.s $x \neq \pi \cdot n$

$\cot(x) \cdot \tan(x) = 1$

hvor begge er definert

$x \neq \frac{\pi}{2} \cdot n \qquad n \in \mathbb{Z}$

cot beskrevet ved tan.

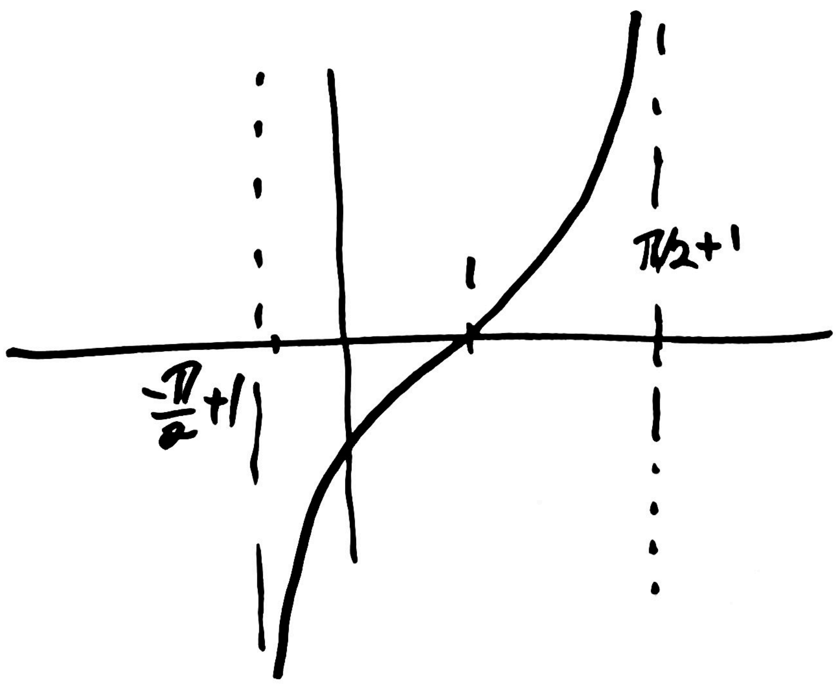
$$\cot(x) = \frac{\cos x}{\sin x} = \frac{\sin(\frac{\pi}{2} - x)}{\cos(\frac{\pi}{2} - x)}$$

$$\underline{\cot(x) = \tan\left(\frac{\pi}{2} - x\right)}$$

tan a) odde funksjon

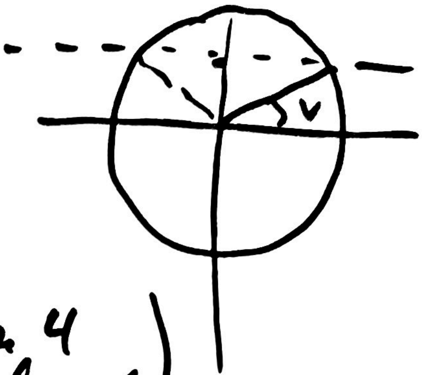
$$\underline{\cot(x) = -\tan\left(x - \frac{\pi}{2}\right)}$$

Lag en skisse av $2\tan(x-1)$



11.7 Trigonometriske ulikheter.

Likning $\sin x = \frac{1}{2}$



$V_1 = \arcsin\left(\frac{1}{2}\right) = \pi/6$
(gir vinkel i 1 eller 4 kvadrant)

mellom $-\frac{\pi}{2}$ og $\frac{\pi}{2}$.

$V_2 = \pi - V_1$ (reflekterer om y-aksen)
$= 5\pi/6$

Legger til hele omløp.

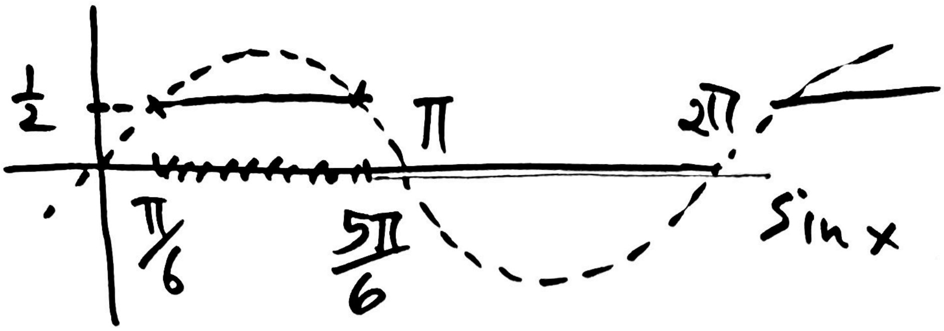
$V = \frac{\pi}{6} + 2\pi \cdot n$

og $V = \frac{5\pi}{6} + 2\pi \cdot n$

Ulikhet $\sin V > \frac{1}{2}$

$V \in [0, 2\pi]$

Ser at løsningene blir $V \in \left\langle \frac{\pi}{6}, \frac{5\pi}{6} \right\rangle$



$$\sin(\underbrace{2x-1}_{u}) \le \frac{1}{2}$$

1. Løser først for u.
2. Løser for x.

1 opp til helt omløp.

$$\underline{\left[\frac{5\pi}{6}, \quad 2\pi+\frac{\pi}{6}\right]}$$

2 $\frac{5\pi}{6}+2\pi\cdot n < 2x-1 < 2\pi+\frac{\pi}{6}+2\pi\cdot n$

legger til 1 på hver side

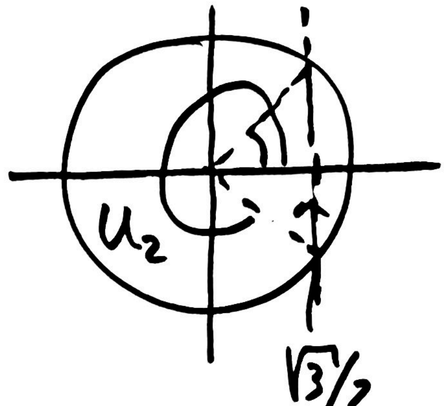
deler med 2 (>0)

$$\underline{\frac{5\pi}{12}+\frac{1}{2}+\pi\cdot n < x < \pi+\frac{\pi}{12}+\frac{1}{2}+\pi\cdot n}$$

$n \in \mathbb{Z}$

$$\cos(3x) < \frac{\sqrt{3}}{2}$$

$x \in [0, \pi].$



$$\cos(u) < \frac{\sqrt{3}}{2}$$

u_1 $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

andre løsnin $-\frac{\pi}{6}$

$$u_2 = -\frac{\pi}{6} + 2\pi = \underline{\frac{11\pi}{6}}$$

$$3x = u$$

$$x = \frac{u}{3}.$$

$$u \in \left\langle \frac{\pi}{6} + 2\pi \cdot n, \ \frac{11\pi}{6} + 2\pi \cdot n \right\rangle$$

$$x \in \left\langle \frac{\pi}{18} + \frac{2\pi}{3} \cdot n, \ \frac{11\pi}{18} + \frac{2\pi}{3} \cdot n \right\rangle$$

I intervallet $[0, \pi]$ er løsningene:

$$\underline{\left\langle \frac{\pi}{18}, \frac{11\pi}{18} \right\rangle \cup \left\langle \frac{\pi}{18} + \frac{2\pi}{3}, \pi \right\rangle}$$