2 mars 2022

15.3 – 4 Flere ubestemte integral.

$$(e^x)' = e^x$$

$$3^x = (e^{\ln 3})^x = e^{x \ln 3}$$

$$(3^x)' = (e^{x \cdot \ln(3)})' = e^{x \ln(3)} \cdot (x \ln 3)'$$

$$= \ln(3)\, 3^x$$

generelt $\quad (a^x)' = \ln(a) \cdot a^x.$

$$\int e^x \, dx = e^x + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + C$$

opg $\quad \int 5^x \, dx = \int e^{\ln 5 \cdot x} \, dx$

$$= \underline{\frac{1}{\ln 5} \cdot 5^x + C}$$

prøver oss frem $\quad (5^x)' = (e^{\ln 5 \cdot x})' = 5^x \cdot \ln 5$

så $\left(\frac{1}{\ln 5} \cdot 5^x\right)' = \frac{1}{\cancel{\ln 5}} \cdot \cancel{\ln 5} \cdot 5^x = 5^x$

opg 1) $\int e^{-3x+1} \, dx$

$$= \frac{1}{-3} e^{-3x+1} + C$$

2) $\int \sqrt{e^{-3x}} \, dx$

$$= \int (e^{-3x})^{1/2} \, dx = \int e^{-3x/2} \, dx$$

$$= \frac{1}{-3/2} e^{-3x/2} + C$$

$$= \underline{\frac{-2}{3} e^{-3x/2} + C}$$

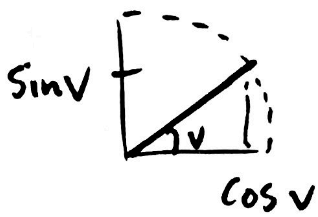
$\int e^{-x^2} \, dx$ Finnes,

men de antideriverte er ikkje en elementær funksjon

(kan ikke uttrykkes ved $\sin x$, e^x, $\ln x$ etc)

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$



$$\int 3\sin(2x+1)\,dx$$

$$= 3\int \sin(2x+1)\,dx$$

$$= 3\left(-\cos(2x+1)\right)\cdot\frac{1}{2} + c$$

$$= \underline{\frac{-3}{2}\cos(2x+1) + c}$$

opg

$$\int 4\cos(\pi x+3)\,dx$$

$$4\int \cos(\pi x+3)\,dx$$

$$= 4\sin(\pi x+3)\cdot\frac{1}{\pi} + c$$

$$= \underline{\frac{4}{\pi}\sin(\pi x+3) + c}$$

$$\int \sin x \, dx = -\cos x + c$$
$$\int \cos x \, dx = \sin x + c$$

$$\int \cos^2 x \, dx$$

Dobling av vinkel $\cos 2x = \cos^2 x - \sin^2 x$

Pytagoras $\cos^2 x + \sin^2 x = 1$

$$\cos(2x) = \cos^2 x - (1 - \cos^2 x)$$
$$= 2\cos^2 x - 1$$

Så $\cos^2 x = \frac{1}{2}(\cos(2x) + 1)$

$$\int \cos^2 x \, dx = \frac{1}{2}\int \cos(2x) + 1 \, dx$$
$$= \frac{1}{2}\left(\frac{\sin(2x)}{2} + x\right) + c$$
$$= \underline{\frac{\sin(2x)}{4} + \frac{x}{2} + c}$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' \quad \text{kvotientregel}$$

$$= \frac{(\sin x)' \cos x - (\sin x)(\cos x)'}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \underline{\frac{1}{\cos^2 x} = 1 + \tan^2 x}$$

$$\int \frac{1}{\cos^2 x}\, dx = \int 1 + \tan^2 x\, dx$$

$$= \underline{\tan(x) + C}$$

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx$$

$$= \int -\frac{(\cos x)'}{\cos x}\, dx$$

$$= \underline{-\ln|\cos x| + C}$$

$$\left(\text{fordi:} \quad (\ln|f(x)|)' = \frac{f'(x)}{f(x)}\right)$$

1. $2\int \frac{1}{4x+3}\,dx$

$= 2\ln|4x+3| \cdot \frac{1}{4} + c$

$= \frac{1}{2}\ln|4x+3| + c$

2. $\int t\sqrt{t+2}\,dt$

$\int ((t+2)-2)\sqrt{t+2}\,dt$

$= \int (t+2)^{3/2} - 2(t+2)^{1/2}\,dt$

$= \frac{(t+2)^{5/2}}{5/2} - 2\frac{(t+2)^{3/2}}{3/2} + c$

$= \frac{2(t+2)^{5/2}}{5} - \frac{4(t+2)^{3/2}}{3} + c$

opg. $\int \tan^2 x \, dx$

$\int (\tan^2 x + 1) - 1 \, dx$

$= \int \tan^2 x + 1 \, dx + \int -1 \, dx$

$= \underline{\tan x \quad - \quad x + C}$

opg 1) $\int \frac{2}{4x+3} \, dx$

2) $\int t\sqrt{t+2} \, dt$

3) $\int \frac{x}{2x+3} \, dx$

4) $\int 2\sin x \cdot \cos x \, dx$

5) $\int \frac{\sin x}{\cos^2 x} \, dx$

$$\int \frac{x}{2x+3}\,dx$$

First pol. division

$$x : 2x+3 = \frac{1}{2} + \frac{-3/2}{2x+3}$$

$$\begin{array}{l} x \\ \underline{x+3/2} \\ \quad -3/2 \end{array}$$

$$\int \frac{x}{2x+3}\,dx = \int \frac{1}{2} - \frac{3}{2}\cdot\frac{1}{2x+3}\,dx$$

$$= \frac{x}{2} - \frac{3}{2}\,\frac{1}{2}\ln|2x+3| + c$$

$$= \underline{\frac{x}{2} - \frac{3}{4}\ln|2x+3| + c}$$

4. $$\int 2\sin x \cdot \cos x \,dx$$

$$= \int \sin(2x)\,dx$$

$$= \underline{-\frac{1}{2}\cos(2x) + C}$$

5. $$\int \frac{\sin x}{\cos^2 x}\,dx = \frac{1}{\cos x} + C$$