4. mars 2022

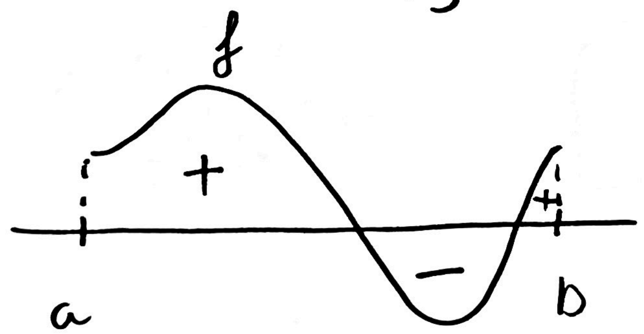
15.5 Bestemte integraler

(1)

$$\int_a^b f(x)\,dx$$

"integralet fra a til b av $f(x)$ med hensyn til x"

$\int_a^b f(x)\,dx$ = areal med fortegn av området mellom grafen til $f(x)$ og x-aksen fra $x=a$ til $x=b$.



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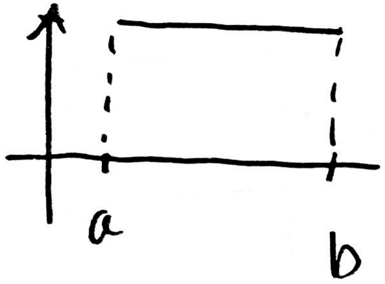
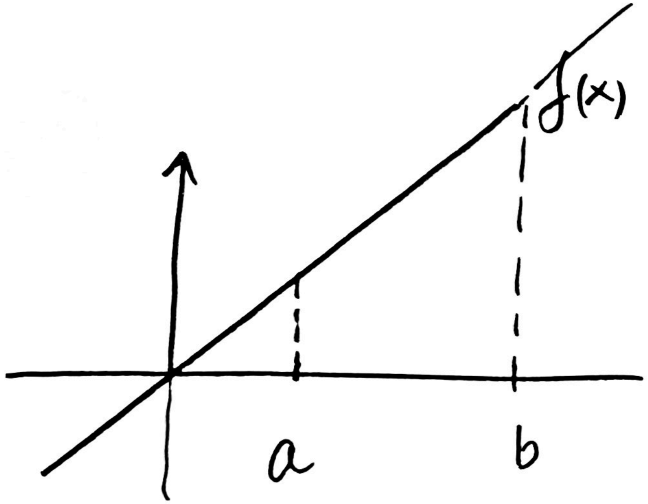


konstant funksjon med verdi k

$$\int_a^b k\,dx = \underline{k(b-a)}$$

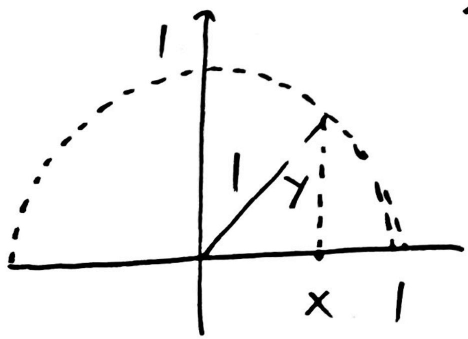
$k \in \mathbb{R}$



$f(x) = k \cdot x$

$$\int_a^b k\,x\,dx = \frac{k \cdot b \cdot b}{2} - \frac{k \cdot a \cdot a}{2}$$

$$= \underline{\frac{k}{2}\left(b^2 - a^2\right)}$$



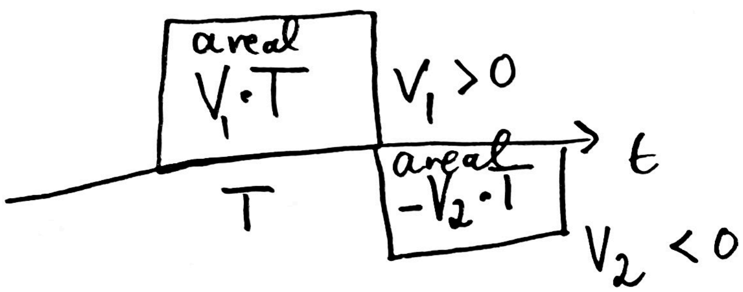
$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{\pi}{2}$$

halvsirkel arealet til en sirkel med radius 1.

3

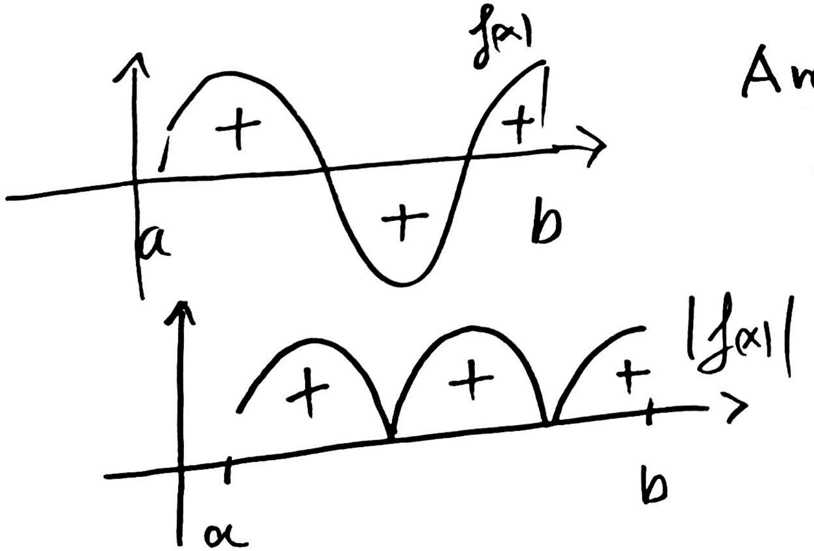


Forflytning langs x-aksen

$$S = V_1 \cdot T + V_2 \cdot T$$

areal over t-akse − areal under t-aksen.

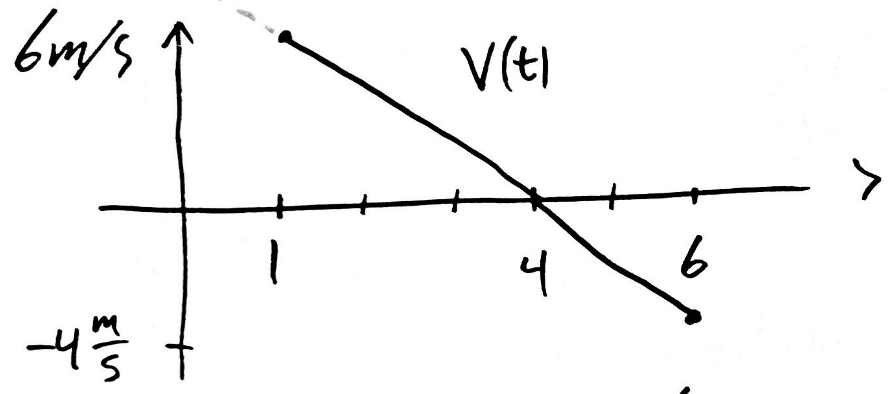
$$S = \int_{t_1}^{t_2} V(t)\, dt$$



Areal mellom grafen til $f(x)$ og x-aksen

$$\int_a^b |f(x)|\, dx$$

(4) Total distanse forflyttet $\int_{t_1}^{t_2} |V(t)|\,dt$

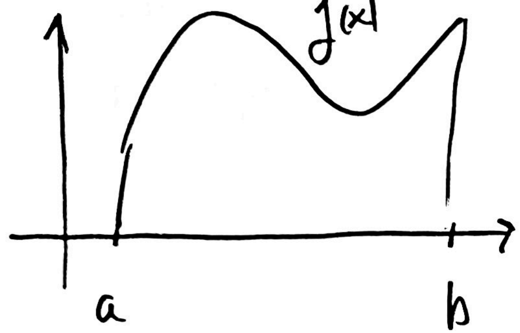


$$V(t) = 6\frac{m}{s} + -2\frac{m}{s^2}(t-1) = 6 - 2(t-1) = \underline{8 - 2t}$$

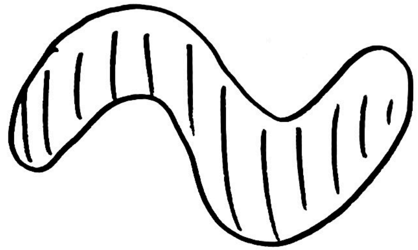
Forflytning $\int_1^6 V(t)\,dt = \frac{6\cdot 3}{2} - \frac{4\cdot 2}{2} = 9 - 4 = \underline{5}$ m

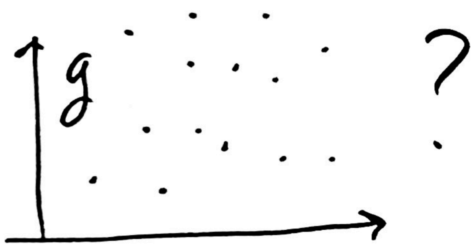
Total distanse $\int_1^6 |V(t)|\,dt = \frac{6\cdot 3}{2} + \frac{4\cdot 2}{2} = 9 + 4 = \underline{13}$ m

5



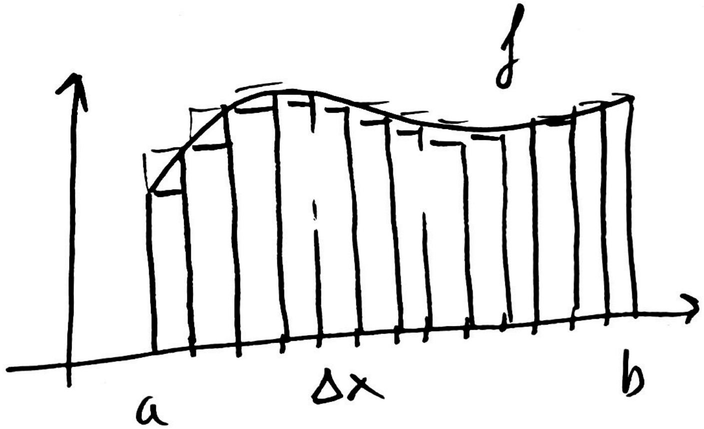
Hva er areal av et område i planet?





problematisk å snakke om areal under grafen til g(x)

Antar f(x) er kontinuerlig



Deler [a, b] i n like intervalle bredden $\Delta x = \frac{b-a}{n}$

⑥ $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \overbrace{\sum_{i=0}^{n-1} \Delta x \cdot f(x_i)}^{S_n}$$

$$x_i \in [a + i\cdot\Delta x,\ a + (i+1)\cdot\Delta x]$$

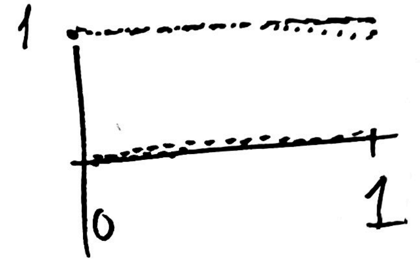
Riemann integral.

Resultat: siden f er kontinuerlig er grensen $n \to \infty$ uavhengig av valgene av x_i.

$$f(x) = \begin{cases} 1 & x \text{ rasjonalt tall} \\ 0 & \text{ellers} \end{cases}$$

$f(\sqrt{2}) = 0$

$f\left(\frac{13}{85}\right) = 1$

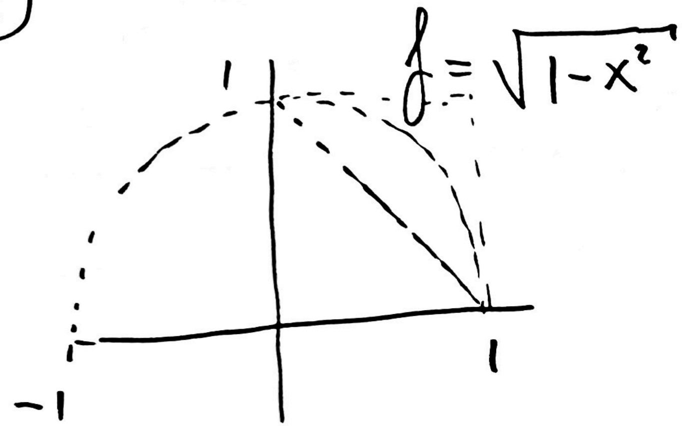


$$\int_0^1 f(x)\,dx = 0$$

Geogebra Lowersum

Oppersum . . .

7

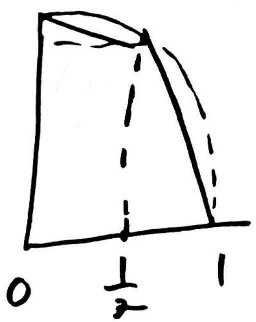


$$\int_{-1}^{1} f(x)\,dx = 2\int_0^1 \sqrt{1-x^2}\,dx$$

Vi $\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4} \sim 0.785\ldots$

$n=1$ Areal trekant $\frac{1 \cdot 1}{2} = 0.5$

$n=2$



$$S_2^{\text{trapes}} = \frac{1}{2}\left(\frac{f(0)+f(\frac{1}{2})}{2} + \frac{f(\frac{1}{2})+f(1)}{2}\right)$$

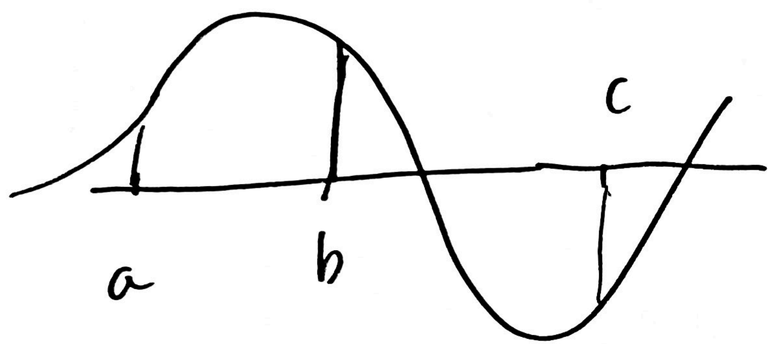
$$= \frac{1}{2}\left(\frac{f(0)}{2} + f(\tfrac{1}{2}) + f(1)\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \sqrt{1-\tfrac{1}{4}} + 0\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \underline{\frac{1}{4}(1+\sqrt{3})} \sim 0.683$$

8

Egenskaper til bestemte integraler



$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

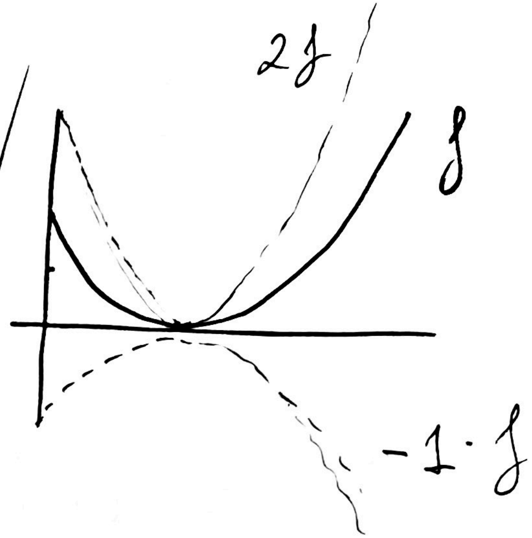
$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^a f(x)\,dx = 0$$

$$\boxed{\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx}$$

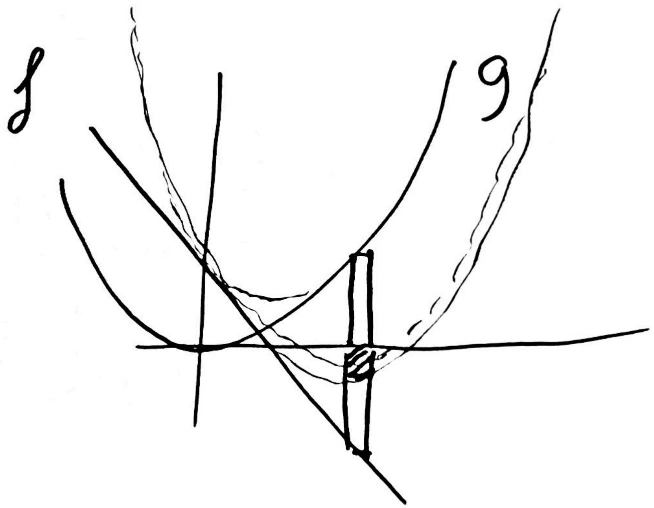
(9)

$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx, \quad k\in\mathbb{R}$$

$$\int_a^b f+g\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$



Bestemte integraler er lineære.



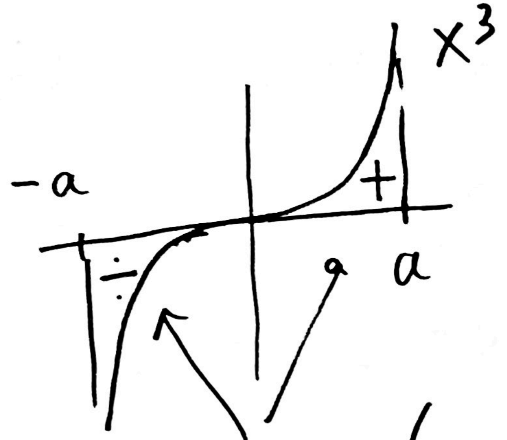
9/9

$$\int_{-1}^{1} 3\sqrt{1-x^2} + 4\,dx$$

$$= 3\int_{-1}^{1}\sqrt{1-x^2}\,dx + \int_{-1}^{1} 4\,dx = 3\cdot\frac{\pi}{2} + 4(1-(-1))$$

$$= \underline{\frac{3\pi}{2} + 8}$$

10



kanselerer hverandre!

$f(x)$ odde funksjon

$a > 0$ $\int_{-a}^{a} x^3 \, dx = 0$

$$\int_{-a}^{a} f(x) \, dx = 0$$

$$\int_{-1}^{1} \underbrace{\sin(x^3)}_{\text{odde funksjon}} \, dx = 0$$

Øving Fredag

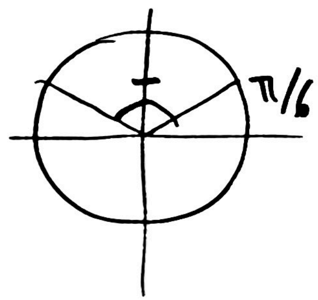
11.90 $f(x) = x + \cos(2x) \quad x \in \langle 0, 2\pi \rangle$

a) topp- og bunnpunkt.

$f'(x) = 1 - \sin(2x) \cdot (2x)' = 1 - 2\sin(2x)$.

$f'(x) = 0$

$\underline{\sin(2x) = \frac{1}{2}}$



$2x = \frac{\pi}{6} + 2\pi \cdot n$

$\frac{5\pi}{6} + 2\pi \cdot n$

deler med 2...

$x = \frac{\pi}{12} + \pi \cdot n \qquad n = 0, 1$

$= \frac{5\pi}{12} + \pi \cdot n$

$f''(x) = -4\cos(2x)$

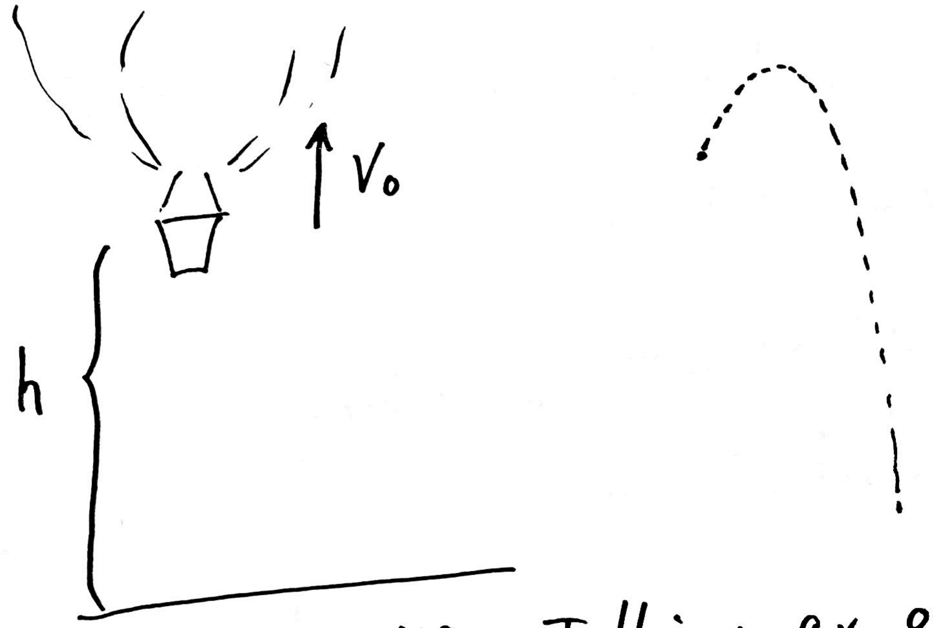
$f''\left(\frac{\pi}{12} + \pi n\right) < 0$

$f''\left(\frac{5\pi}{12} + \pi n\right) > 0$

Toppunkt i $\left(\frac{\pi}{12}, \frac{\pi}{12}+\frac{\sqrt{3}}{2}\right)$, $\left(\frac{13\pi}{12}, \frac{13\pi}{12}+\frac{\sqrt{3}}{2}\right)$

Bunnpunkt i $\left(\frac{5\pi}{12}, \frac{5\pi}{12}-\frac{\sqrt{3}}{2}\right)$, $\left(\frac{17\pi}{12}, \frac{17\pi}{12}-\frac{\sqrt{3}}{2}\right)$

Øving
Torsdag
3. mars



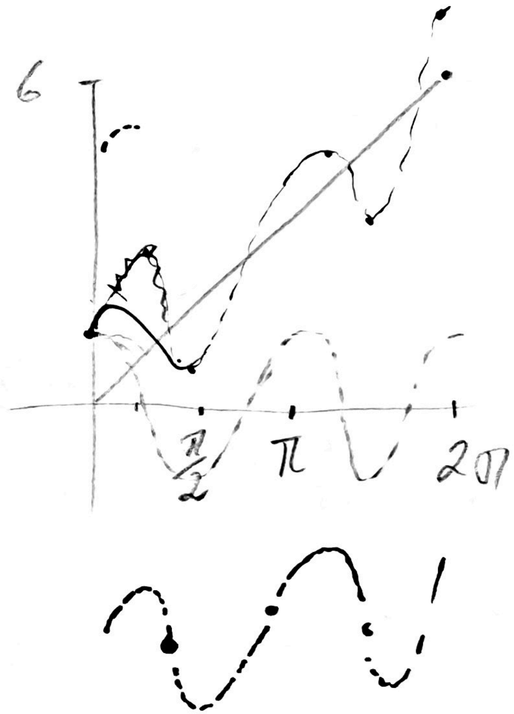
opg 15.118 Tolking av opg.

11.90 Funksjonsdrøfting av trig. funksjoner

$f(x) = x + \cos(2x)$ $\langle 0, 2\pi \rangle$

$f'(x) = 1 - \sin(2x) \cdot (2x)'$

$= 1 - 2\sin(2x)$



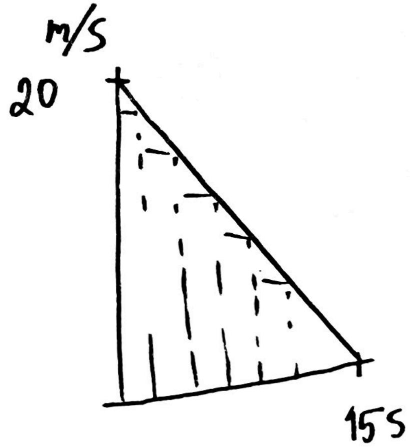
Så $f'(x) = 0$

$\sin(2x) = \frac{1}{2}$

$f''(x) = -2\cos(2x) \cdot (2x)' = -4\cos(2x)$

f' skifter fortegn når

$f''(x) = 0 \iff \cos(2x) = 0$



a) akselerasjon a

b) S distanse forflyttet under oppbremsing.

$V(t) = 20 - \frac{4}{3}t$. $D_V = [0, 15]$

$a(t) = V'(t)$

$S'(t) = V(t)$ $S(0) = 0$

$S(15)$ forflytningen.

$S(15)$ = areal under $V(t)$ fra $t=0$ til 15.

$= \frac{20 \cdot 15}{2} = 150$ meter.

15.135 $\left(\dfrac{e^x}{e^x+1}\right)' = \left(e^x \cdot \dfrac{1}{e^x+1}\right)'$

$$= \frac{(e^x)'(e^x+1) - e^x(e^x+1)'}{(e^x+1)^2}$$

$$= \frac{e^x(e^x+1) - e^x(e^x)}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2}$$

b) så $\displaystyle\int \frac{e^x}{(e^x+1)^2}\,dx = \frac{e^x}{e^x+1} + C$

15.124 $\left(\ln(x^2+1)\right)' \underset{\text{kjerneregel}}{=} \dfrac{1}{x^2+1} \cdot \underbrace{(x^2+1)'}_{2x}$

$$= \frac{2x}{x^2+1}$$