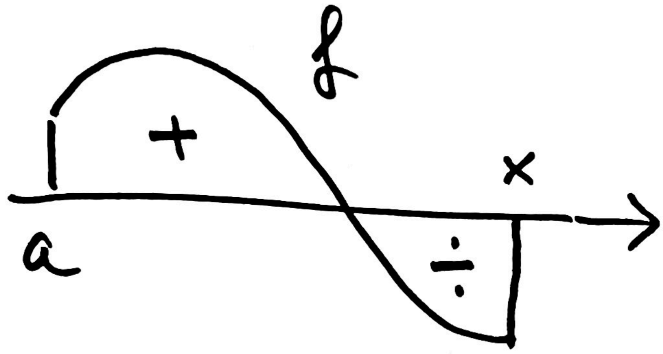
15.7 integraler og areal.

9.03
2022



$$F(x) = \int_a^x f(t)\,dt$$

= areal med fortegn av regionen avgrenset av grafen til f og x-aksen.

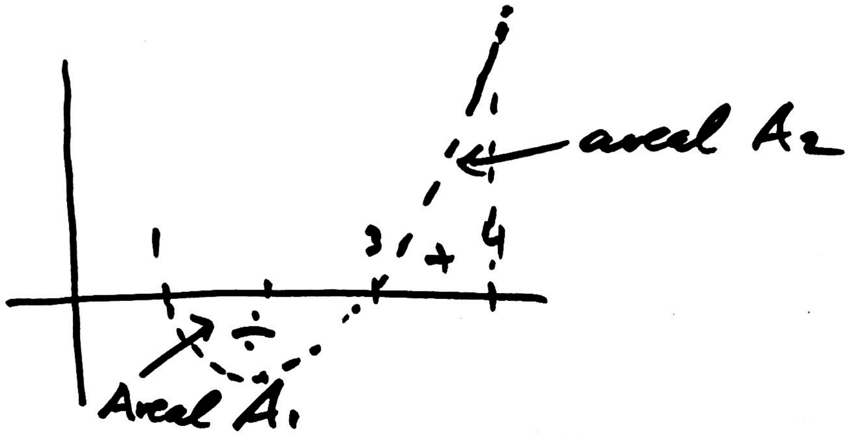
$$F'(x) = \frac{d}{dx} F(x) = f(x)$$

når f er en kontinuerlig funksjon.

⇒ For enhver antiderivert $G(x)$ til $f(x)$ så er

$$\int_a^b f(x)\,dx = G(b) - G(a)$$
$$= G(x)\Big|_a^b$$

Finn areal m. fortegn mellom grafen til $(x-2)^2-1$ og x-aksen fra $x=1$ til $x=4$.



$$\int_1^4 (x-2)^2 - 1 \, dx$$

$$\left.\frac{(x-2)^3}{3} - x \right|_1^4$$

$$= \frac{2^3}{3} - \frac{(-1)^3}{3} - 4 - (-1)$$

$$= \frac{8+1}{3} - 3 = \frac{9}{3} - 3 = \underline{0}$$

$-A_1 + A_2 = 0$ så $A_1 = A_2$.

Hva er arealet mellom grafen til $f(x)$ og x-aksen fra $x=1$ til $x=4$?

$$A_1 + A_2$$

$$\int_1^4 |f(x)| \, dx = \underbrace{-\int_1^3 f(x)\,dx}_{A_1} + \underbrace{\int_3^4 f(x)\,dx}_{A_2}$$

$$A_1 = -\int_1^3 f(x)\,dx = -\int_1^3 (x-2)^2 - 1\,dx$$

$$= -\left(\frac{(x-2)^3}{3} - x\right)\Bigg|_1^3$$

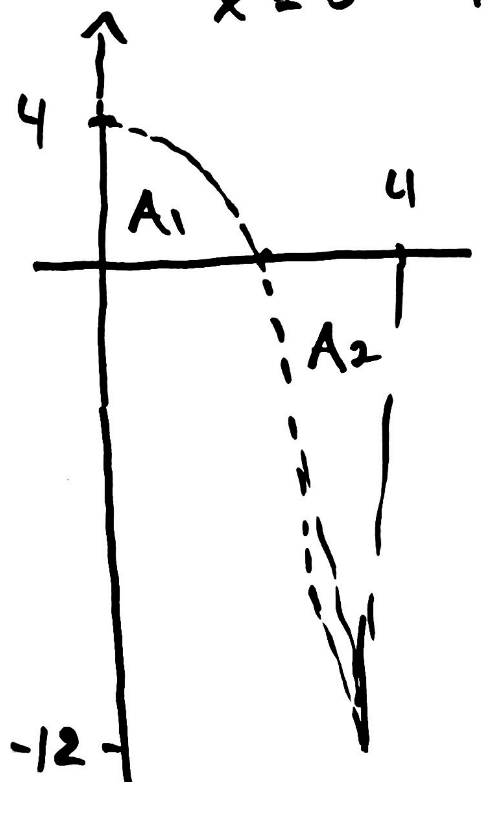
$$= -\left(\frac{1}{3} - \frac{-1}{3} - 3 - (-1)\right)$$

$$= -\left(\frac{2}{3} - 2\right) = -\left(\frac{-4}{3}\right)$$

$$A_1 = A_2 = \frac{4}{3}$$

$$\int_1^4 |f(x)|\,dx = A_1 + A_2 = 2 \cdot \frac{4}{3} = \underline{\frac{8}{3}}$$

Finn arealet mellom grafen til $4 - x^2$ og x-aksen fra $x = 0$ til $x = 4$



$$A = A_1 + A_2$$

Nullpunkt:

$$4 - x^2 = 0$$

$$x^2 = 4$$

$x = \underline{2}$ i $[0, 4]$

$$A_1 = \int_0^2 4 - x^2 \, dx$$

$$A_2 = - \int_2^4 4 - x^2 \, dx$$

$$\int 4 - x^2 \, dx = 4x - \frac{x^3}{3} + C$$

$$A_1 = \int_0^2 4 - x^2 \, dx = 4x - \frac{x^3}{3} \Big|_0^2$$

$$= 4 \cdot 2 - \frac{2^3}{3} = 8 - \frac{8}{3}$$

$$= \underline{\frac{16}{3}}$$

$$A_2 = - \int_2^4 4 - x^2 \, dx$$

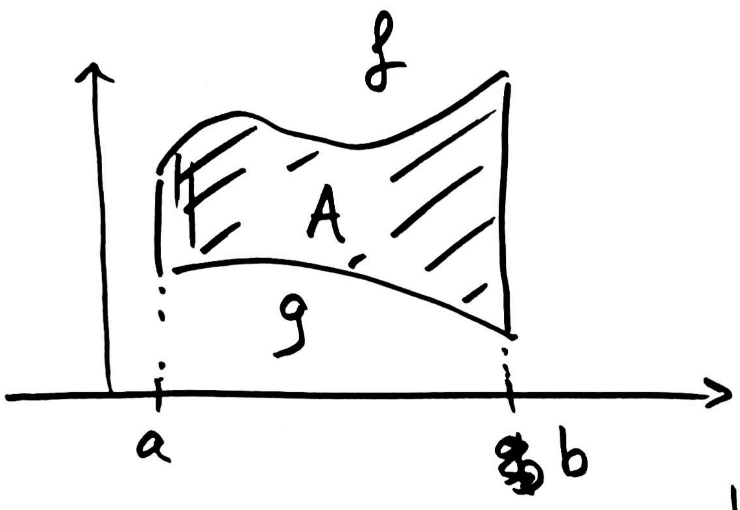
$$= - \left(4x - \frac{x^3}{3}\right)\Big|_2^4$$

$$= \frac{16}{3} + - \underbrace{\left(4 \cdot 4 - \frac{4^3}{3}\right)}_{4^2\left(1 - \frac{4}{3}\right)}$$

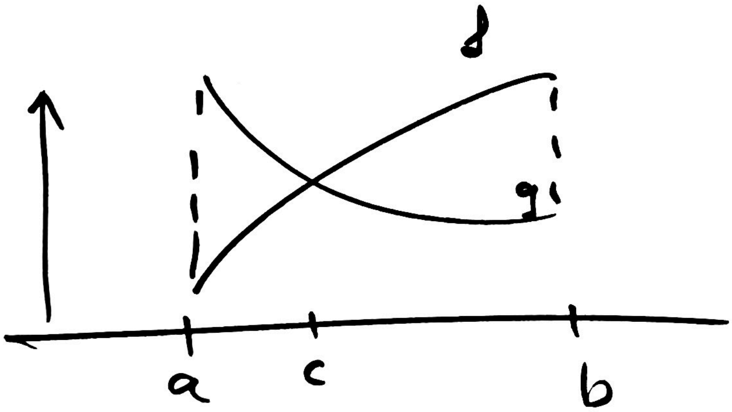
$$= \frac{16}{3} + \frac{16}{3} = \underline{\frac{32}{3}}$$

$$A_1 + A_2 = \frac{16}{3} + \frac{2 \cdot 16}{3} = \underline{16}$$

$f \geq g$



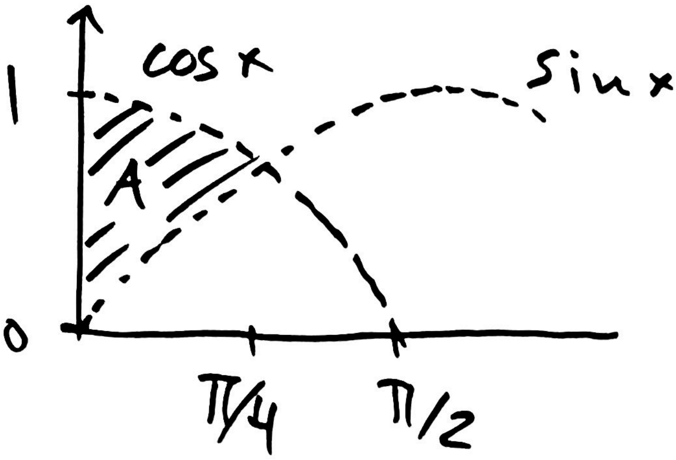
$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$
$$= \int_a^b \left(f(x) - g(x)\right) dx$$

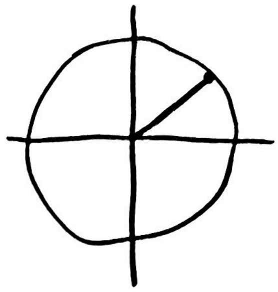


$$\int_a^b \left|(f-g)(x)\right| dx$$

Til figuren ovenfor:

$$\int_a^c g(x) - f(x)\,dx$$
$$+ \int_c^b f(x) - g(x)\,dx \,.$$





$\cos x = \sin x$

$x = \frac{\pi}{4}$

i $[0, \frac{\pi}{2}]$

$$A = \int_0^{\pi/4} \cos x - \sin x \, dx$$

$$1 = \tan x$$

$$x = \arctan 1 + \pi \cdot n$$

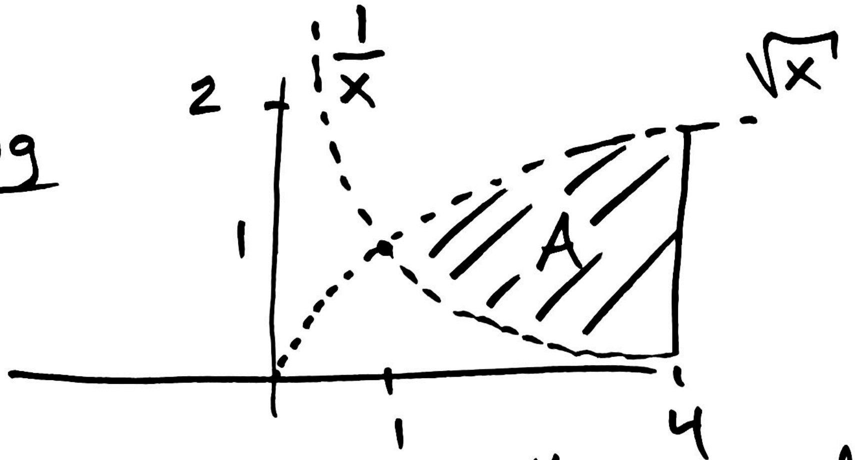
etc

$$= \sin x + \cos x \Big|_0^{\pi/4}$$

$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos(0)$$

$$= 2 \cdot \frac{1}{\sqrt{2}} - 1$$

$$= \underline{\underline{\sqrt{2} - 1}} \sim 0.41$$



opg

Finn arealet mellom grafen til $\frac{1}{x}$ og grafen til $\sqrt{x}$ fra $x = 1$ til $x = 4$.

$$A = \int_1^4 \sqrt{x} - \frac{1}{x}\, dx$$

$$= \int_1^4 x^{1/2} - \frac{1}{x}\, dx$$

$$= \frac{x^{3/2}}{3/2} - \ln|x| \Big|_1^4$$

$$= \frac{2}{3}\Big(\underbrace{4^{3/2}}_{4\sqrt{4}} - 1^{3/2}\Big) - \big(\ln 4 - \underbrace{\ln 1}_{0}\big)$$

$$= \frac{2}{3}(8 - 1) - \ln 4$$

$$= \underline{\frac{14}{3} - \ln 4} \sim \underline{3.28}$$

15.8 samla resultat

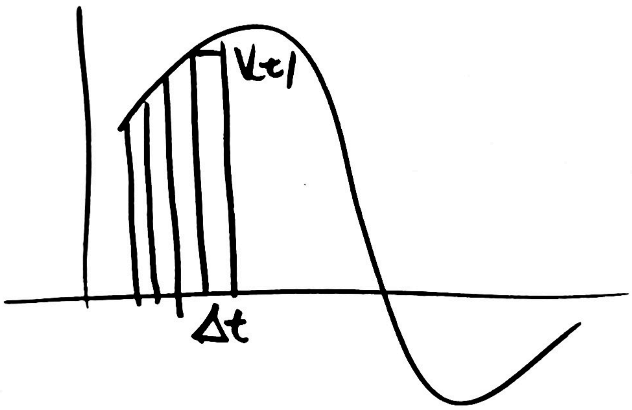
v konstant

Distanse $s = v \cdot t$

hastighet · tid

$$s = \int_0^t v(t)\, dt$$

v varierer med tiden.



$v = a \cdot t$

akselerasjon

$$v = \int a(t)\, dt.$$

a konstant gir bevegelses likningene

$$v(t) = \int_0^t a\, dt$$
$$= a \cdot t$$

$V(0) = 0$ ser på tilfellet $V(0) = V_0$.

$$V(t) = a \cdot t + V_0$$

$$S(t) - S(0) = \int_0^t V(\sigma)\, d\sigma$$

$$= \int_0^t a \cdot \sigma + V_0 \, d\sigma$$

$$= \frac{1}{2} a t^2 + V_0 t$$

Bevegelseslikningen

$$S(t) = \frac{1}{2} a t^2 + V_0 t + S_0 .$$

<u>eks</u> $V_0 = 0$, $S_0 = 0$

$$a = 1 - t \qquad t \in [0, 1].$$

Hva er forflyttingen fra $t = 0$ til $t = 1$?

$$V(t) = \int_0^t a(\sigma)\, d\sigma$$

$$= \int_0^t 1 - \sigma \, d\sigma$$

$$= \left(\sigma - \frac{\sigma^2}{2}\right)\Big|_0^t = \cancel{\;}.$$

$$V(t) = \underline{t - \frac{t^2}{2}}$$

$$S(t) = \int_0^t V(s)\, ds$$

$$= \int_0^t s - \frac{s^2}{2}\, ds$$

$$\frac{s^2}{2} - \frac{s^3}{6} \Big|_0^t$$

$$\underline{S(t) = \frac{t^2}{2} - \frac{t^3}{6}}$$

Kraft · veg = Energi.

$$F = m \cdot V'$$

$$\Delta x = V \cdot \Delta t$$

$$\int F \cdot dx = \int F \cdot V\, dt$$

$$= m \int \underbrace{V' \cdot V}_{\left(\frac{V^2}{2}\right)'} dt$$

$$\underline{E = \frac{m}{2} \cdot V^2}$$