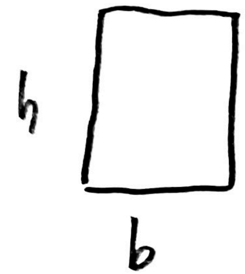
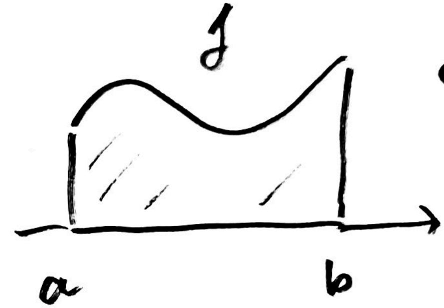
16.1 Volum og integrasjon.

15 mars 2022

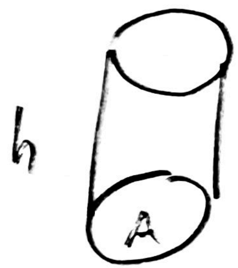


areal $b \cdot h$

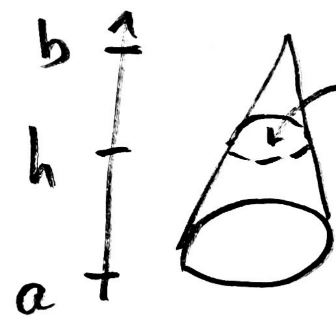


$f > 0$

Areal $\int_a^b f(x)\,dx$



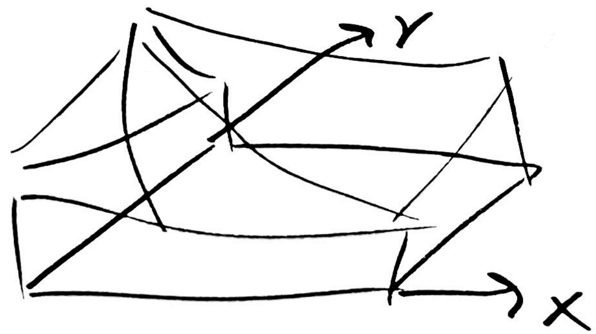
volumet $A \cdot h$



tverrsnitt areal $A(h)$

$$V = \int_a^b A(h)\,dh$$

Volum

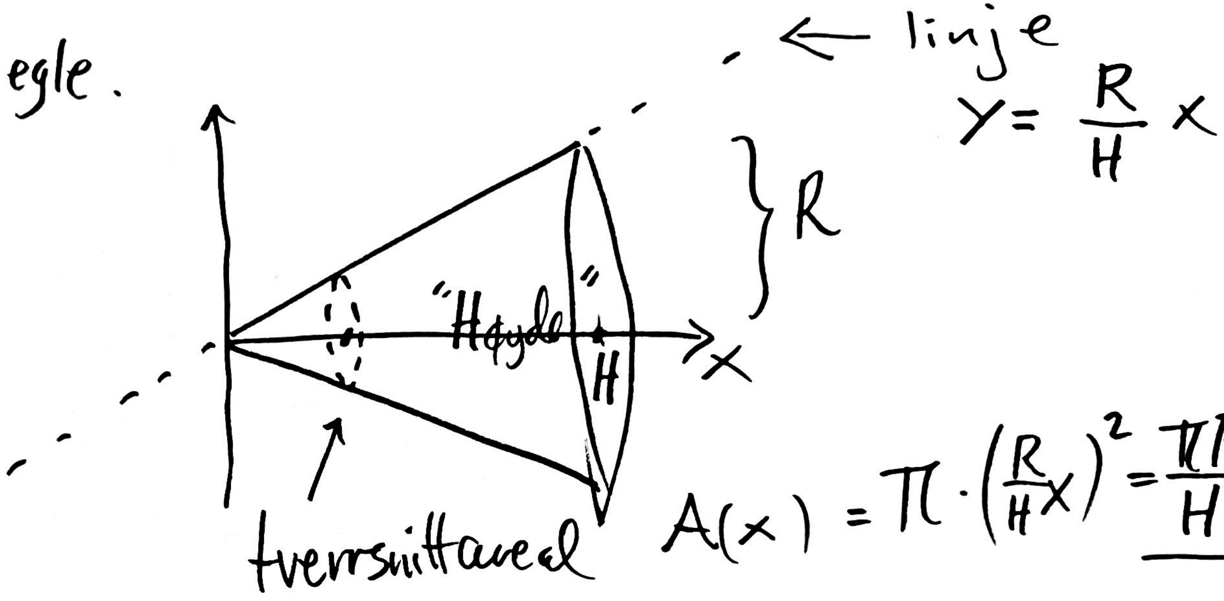


$h(x,y)$

$$\iint_A h(x,y)\,dx\,dy$$

(M2000)

Voum til en kjegle.



$A(x) = \pi \cdot \left(\frac{R}{H} x\right)^2 = \underline{\frac{\pi R^2}{H^2} x^2}$

$$V = \lim_{\Delta x \to 0} \sum_i A(x_i) \cdot \Delta x = \int_0^H A(x)\,dx$$

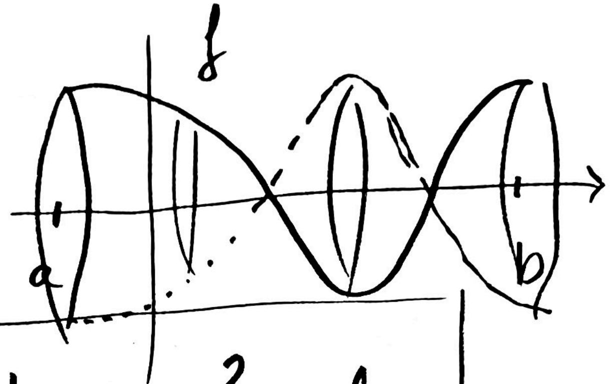
$$V = \int_0^H \frac{\pi R^2}{H^2} \cdot x^2\,dx = \frac{\pi R^2}{H^2} \cdot \frac{x^3}{3} \Big|_0^H = \frac{\pi R^2}{3H^2} \cdot x^3 \Big|_0^H$$

$$= \frac{\pi R^2}{3 H^2}\left(H^3 - 0^3\right) = \underline{\frac{\pi R^2}{3} \cdot H}$$

omdreiningslegeme

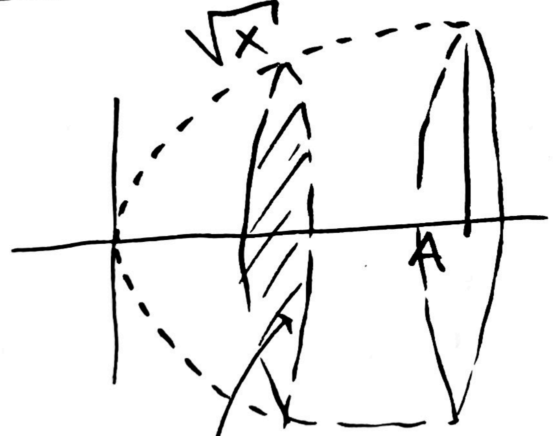
om x-aksen

fra $x=a$ til $x=b$



$$\boxed{V = \int_a^b \pi f(x)^2 \, dx}$$

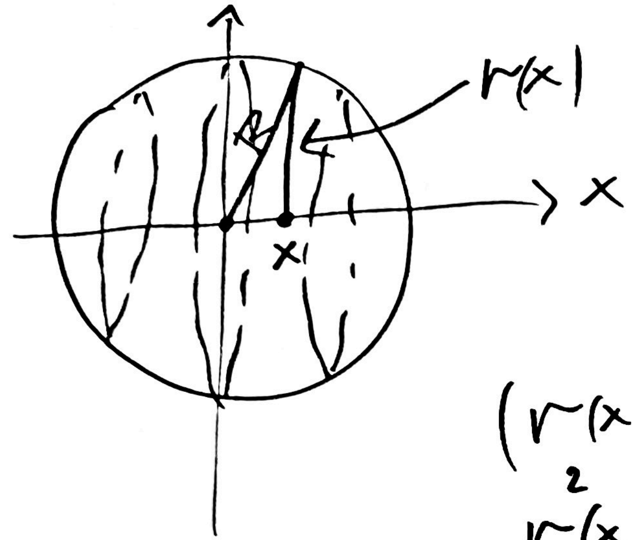
opg



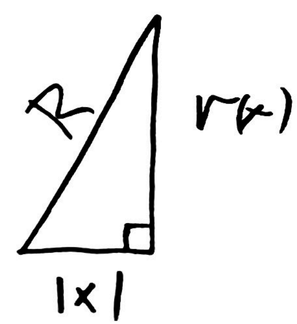
Hva er volumet til omdreiningslegemet begrenset av
$f(x) = \sqrt{x}$
fra $x=0$ til $x=A$.

$A(x) = \pi(\sqrt{x})^2 = \pi x$

$$V = \int_0^A \pi \cdot x \, dx = \pi \cdot \frac{x^2}{2} \Big|_0^A = \frac{\pi}{2}(A^2 - 0^2) = \underline{\underline{\frac{\pi A^2}{2}}}$$



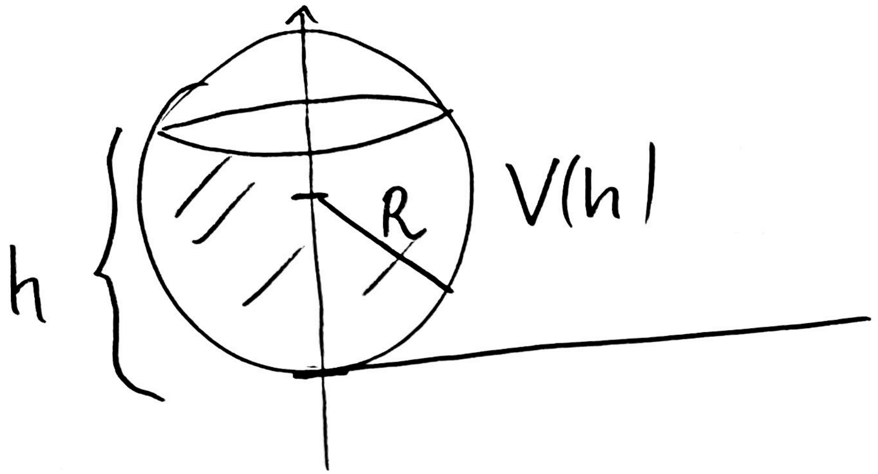
Kule med radius R



$$(r(x))^2 + x^2 = R^2$$
$$r^2(x) = R^2 - x^2$$
$$r(x) = \sqrt{R^2 - x^2}$$

Tverrsnittarealet i x er $A(x) = \pi \cdot r^2_{(x)}$
$$= \pi(R^2 - x^2)$$

$$V = \int_{-R}^{R} A(x)\,dx = 2\int_0^R \pi(R^2 - x^2)\,dx$$
$$= 2\pi\left[R^2 \cdot x - \frac{x^3}{3}\right]\Big|_0^R$$
$$= 2\pi\left[R^2 \cdot R - \frac{R^3}{3} - 0\right] = 2\pi R^3\left(1 - \frac{1}{3}\right) = \underline{\underline{\frac{4\pi}{3} R^3}}$$



Hva er $V(h)$?

La $x = h - R$

$h = x + R$.

$$\int_{-R}^{x} A(y)\,dy = \int_{-R}^{x} \pi(R^2 - y^2)\,dy$$

$$= \pi\left(R^2 y - \frac{y^3}{3}\right)\Big|_{-R}^{x} = \pi\left(R^2 x - \frac{x^3}{3}\right) - \pi\left(-R^3 + \frac{R^3}{3}\right)$$

$$= \pi\left(R^2 x - \frac{x^3}{3}\right) + \frac{2\pi}{3}R^3$$

$$= \pi\left(R^2(h-R) - \frac{(h-R)^3}{3}\right) + \frac{2\pi}{3}R^3$$

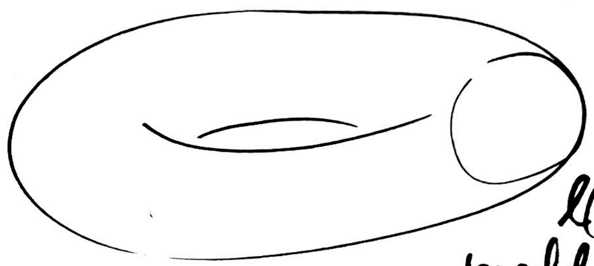
$$= \pi\left(R^2 h - R^3 - \frac{h^3 - 3h^2R + 3hR^2 - R^3}{3}\right) + \frac{2\pi}{3}R^3$$

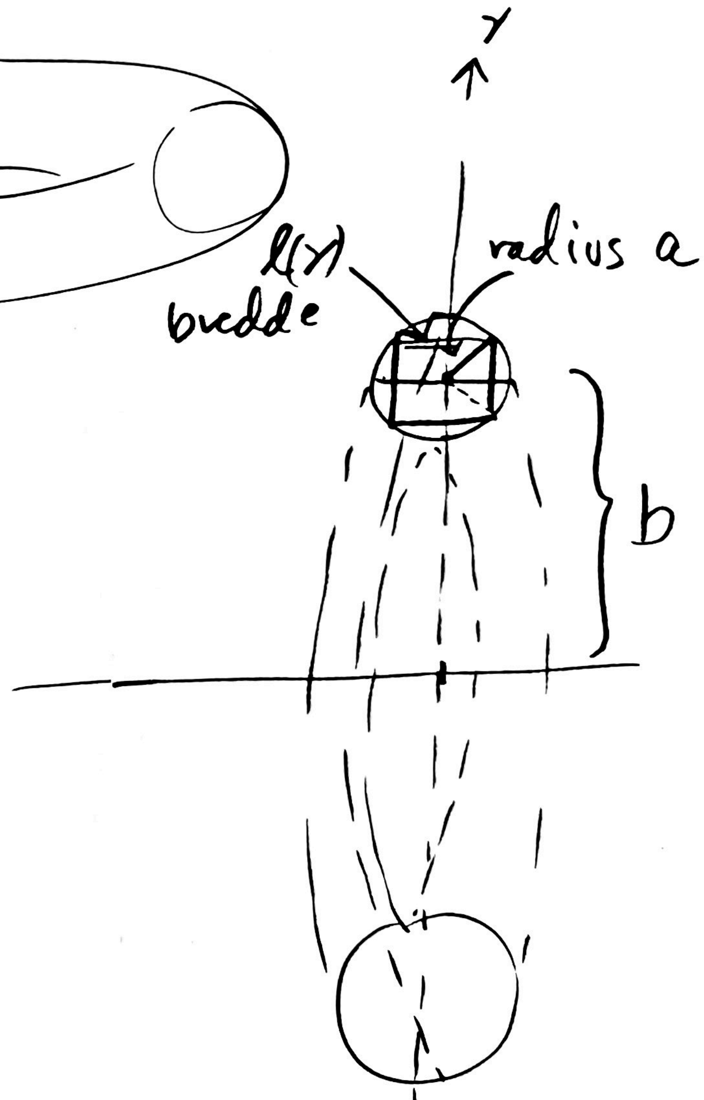
$$= \pi\left(R^3\underbrace{\left(-1 + \frac{1}{3} + \frac{2}{3}\right)}_{0} + R^2 h\underbrace{(1-1)}_{0} + Rh^2(1) - \frac{h^3}{3}\right)$$

$$\underline{V(h) = \pi\left(Rh^2 - \frac{h^3}{3}\right)}$$

Torus

(smultring)





$$b-a \le y \le b+a$$

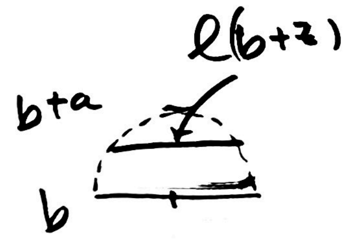
$$V = \int_{b-a}^{b+a} l(y) \cdot 2\pi(y)\, dy$$

$$l(b+z) = l(b-z)$$

$$V = 2\pi \int_0^a \underbrace{l(b+z)}_{\shortparallel} (b+z)\, dz + 2\pi \int_0^a \overbrace{l(b-z)} (b-z)\, dz$$

$$\underline{a \le b}$$

$$V = 2\pi \int_0^a \ell(b+z)\left(\underbrace{\frac{b+z+b-z}{}}_{2b}\right) dz$$

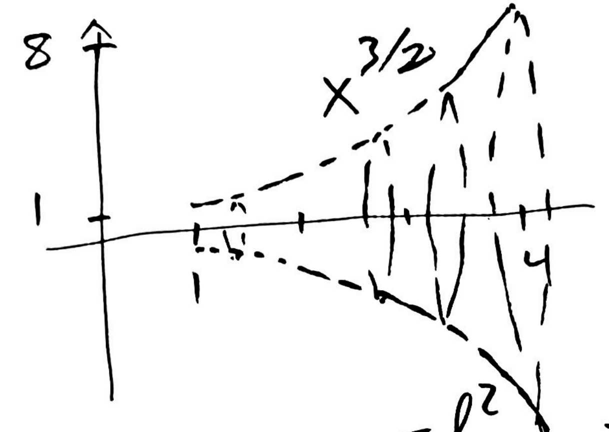


$$= 2\pi \cdot 2b \underbrace{\int_0^a \ell(b+z)\, dz}_{\frac{\pi a^2}{2}}$$

$$\underline{V = 2\pi^2 b \cdot a^2}$$

opg

Finn volumet til legemet som frem kommer ved å rotere området avgrenset av $f(x) = x^{3/2} = x\sqrt{x}$ fra $x = 1$ til $x = 4$



$$A(x) = \pi f_{(x)}^2 = \pi (x^{3/2})^2 = \pi x^3$$

$$V = \int_1^4 A(x)\,dx = \pi \frac{x^4}{4}\Big|_1^4 = \pi\left(\frac{4^4 - 1^4}{4}\right)$$

$$= \int_1^4 \pi x^3\,dx$$

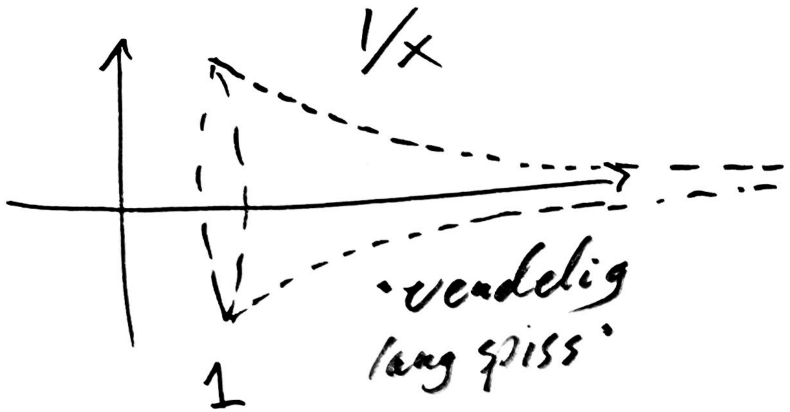
$$= \pi\left(\frac{16^2 - 1}{4}\right) = \underline{\frac{255 \cdot \pi}{4}}$$

$16 \cdot 16$

$= (10+6)(10+6)$

$= 10^2 + 2 \cdot 10 \cdot 6 + 6^2$

$= 256$



Volumet

$$V = \int_1^\infty \pi \cdot \left(\frac{1}{x}\right)^2 dx$$

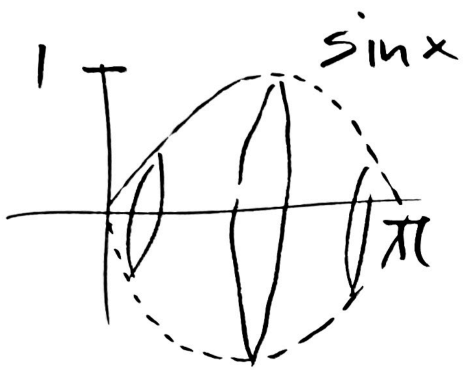
$$= \lim_{R\to\infty} \int_1^R \pi \cdot x^{-2} dx$$

$$= \lim_{R\to\infty} \pi \left(\frac{-1}{x}\right)\Big|_1^R$$

$$= \lim_{R\to\infty} \pi \left(\frac{-1}{R} - \left(\frac{-1}{1}\right)\right) = \lim_{R\to\infty} \pi\left(1 - \frac{1}{R}\right)$$

$$= \pi$$

opg



Volumet

$$V = \int_0^{\pi} \pi \cdot \sin^2 x \, dx$$

$$V = \pi \underbrace{\int_0^{\pi} \sin^2 x \, dx}_{\int_0^{\pi} \cos^2 x \, dx} = \pi \cdot \frac{1}{2} \int_0^{\pi} \overbrace{\sin^2 x + \cos^2 x}^{1} \, dx$$

$$V = \pi \cdot \frac{\pi}{2} = \underline{\underline{\frac{\pi^2}{2}}}$$