22 mars 2022 16.5 Integraler av rasjonale funksjoner.

Delbrøksoppspalting

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$u = 3x-4$

$u' = 3$

$dx = \frac{1}{3}\,du$

$$\int \frac{1}{3x-4}\,dx = \int \frac{1}{u}\frac{1}{3}\,du$$

$$= \frac{1}{3}\ln|u| + C = \underline{\frac{1}{3}\ln|3x-4| + C}$$

$$\int \frac{2x}{x-3}\,dx$$

$2x : x-3 = \underline{2 + \frac{6}{x-3}}$

$\underline{2x-6}$

6

$$= \int \frac{2(x-3)+6}{x-3}\,dx$$

$$= \int 2 + \frac{6}{x-3}\,dx = \underline{2x + 6\ln|x-3| + C}$$

$$\int \frac{1}{(3x-4)^2}\,dx$$

$u = 3x - 4$

$u' = 3$

$$= \int \frac{1}{u^2} \cdot \frac{1}{3}du = \frac{1}{3}\int u^{-2}\,du$$

$$= \frac{1}{3} \cdot \frac{1}{-1} \cdot u^{-1} + C = \underline{\frac{-1}{3(3x-4)} + C}$$

$4x+3 = 4(x-1)+7$

$$\int \frac{4x+3}{(x-1)^2}\,dx$$

$$\int \frac{4(x-1)+7}{(x-1)^2}\,dx = \int \frac{4}{x-1} + \frac{7}{(x-1)^2}\,dx$$

$$= 4\ln|x-1| + 7\left(\frac{-1}{x-1}\right) + C$$

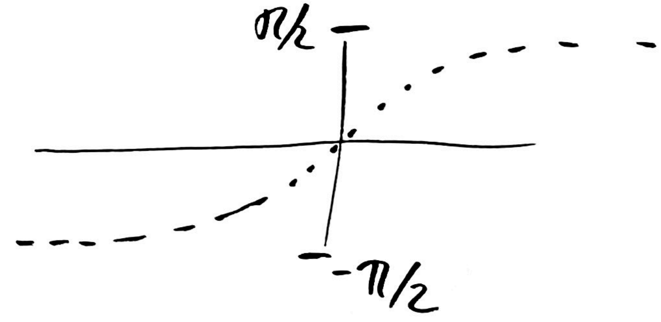
$$= \underline{4\ln|x-1| - \frac{7}{x-1} + C}$$

$$\tan(\arctan x) = x \qquad \text{deriverer}$$

$$\tan'(\arctan x) \cdot (\arctan x)' = (x)' = 1$$

$$\underbrace{1 + \tan^2(\arctan x)}_{(1+x^2)} \cdot (\arctan x)' = 1$$

$$\boxed{(\arctan x)' = \frac{1}{1+x^2}}$$



$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

$$\int \frac{1}{x^2+4x+13}\,dx = \int \frac{1}{(x+2)^2+9}\,dx$$

$$= \int \frac{1/9}{\left(\frac{x+2}{3}\right)^2+1}\,dx$$

$$V = \frac{x+2}{3}$$

$$\frac{dV}{dx} = \frac{1}{3}$$

$$3\,dV = dx$$

$$= \frac{1}{9} \int \frac{1}{v^2+1} \, 3\,dv$$

$$= \frac{3}{9} \arctan v + C$$

$$\int \frac{1}{x^2+4x+13} \, dx = \underline{\frac{1}{3} \arctan\left(\frac{x+2}{3}\right) + C}$$

$$\int \frac{5}{(x+2)(x-3)} \, dx =$$

$$\frac{5}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{(x-3)}$$

ganger med $(x+2)(x-3)$ på begge sider av likhetstegnet

$$5 = A(x-3) + B(x+2) \quad \text{for alle } x.$$

Sette $x=3$

$$5 = B(3+2) = 5B \quad \text{så } \underline{B=1}$$

Sette $x=-2$

$$5 = A(-2-3) = -5A \text{, så } \underline{A=-1}$$

$$\int \frac{5}{(x+2)(x-3)}\,dx = \int \frac{-1}{(x+2)} + \frac{1}{x-3}\,dx$$

$$= -\ln|x+2| + \ln|x-3| + C$$

$$= \underline{\ln\left|\frac{x-3}{x+2}\right| + C}$$

opg $\displaystyle\int \frac{1}{x^2+3x+2}\,dx = \int \frac{1}{(x+2)(x+1)}\,dx$

$$= \int \frac{A}{x+2} + \frac{B}{x+1}\,dx = \int \frac{-1}{x+2} + \frac{1}{x+1}\,dx$$

$$= -\ln|x+2| + \ln|x+1| + C = \underline{\underline{\ln\left|\frac{x+1}{x+2}\right| + C}}$$

$$\frac{1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

Felles nevner og sammenligne tellerne

$$\frac{1}{(x+2)(x+1)} = \frac{A(x+1) + B(x+2)}{(x+2)(x+1)}$$

$$0x + 1 = A(x+1) + B(x+2) = (A+B)x + A + 2B$$

Så $A+B=0$, $A+2B=1$

Så $B = (A+2B) - (A+B) = 1 - 0 = \underline{1}$

$A = -B = \underline{-1}$

oppgave $\int \frac{x^2}{x^2+3x+2}\,dx = \int \frac{(x^2+3x+2) - 3x - 2}{x^2+3x+2}\,dx$

$$= \int 1 - \frac{3x+2}{x^2+3x+2}\,dx$$

$$\frac{3x+2}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$3x+2 = A(x+1) + B(x+2)$$

$x=-1$: $\quad \underline{-1} = B(-1+2) = \underline{B}$

$x=-2$ $\quad -4 = A(-2+1) = -A$,

$A=4$ og $B=-1$

$$\text{Int} = \int 1 - \left(\frac{4}{x+2} - \frac{1}{x+1}\right) dx$$

$$= \underline{x - 4\ln|x+2| + \ln|x+1| + C}$$

$$= \underline{x + \ln\left|\frac{x+1}{(x+2)^4}\right| + C}$$

$$\int \frac{2x+3}{x^2+3x+2}\,dx \qquad u = x^2+3x+2$$

$$u' = 2x+3$$

$$= \int \frac{u'}{u}\,dx$$

$$= \ln|u| + C = \underline{\ln|x^2+3x+2| + C}$$

Eks $\displaystyle\int \frac{2x+5}{(x-1)^2(x+2)}\,dx$

$$\frac{2x+5}{(x-1)^2(x+2)} = \frac{A}{x+2} + \underbrace{\frac{Bx+C}{(x-1)^2}}_{\frac{E}{(x-1)^2} + \frac{D}{(x-1)}}$$

$$2x+5 = A(x-1)^2 + E(x+2) + D(x-1)(x+2)$$

sette $x=1$: $\quad 7 = A\cdot 0 + E\cdot 3 + D\cdot 0$

$E = 7/3$

setter $x = -2$ $\quad 1 = A(-2-1)^2 + 0 = 9A$

$A = 1/9.$

$x = 0$ $\quad 5 = A + 2E + D(-1)(2)$

$D = \frac{1}{2}(A + 2E - 5)$

$= \frac{1}{2}\left(\frac{1}{9} + 2 \cdot \frac{7}{3} - 5\right) = \frac{1}{2} \cdot \frac{1}{9}(1 + 3 \cdot 14 - 5 \cdot 9)$

$= \frac{1}{2 \cdot 9}(1 + 42 - 45) = \underline{\frac{-1}{9}}$

$$\int \frac{2x+5}{(x-1)^2(x+2)}\,dx = \int \frac{1}{9} \cdot \frac{1}{x+2} + \frac{7}{3}\frac{1}{(x-1)^2} - \frac{1}{9}\frac{1}{x-1}\,dx$$

$$= \frac{1}{9}\left(\ln|x+2| - \ln|x-1|\right) + \frac{7}{3}\frac{-1}{(x-1)} + c$$

$$= \frac{1}{9}\ln\left|\frac{x+2}{x-1}\right| - \frac{7/3}{(x-1)} + c$$

$$= \underline{\frac{1}{9}\ln\left|1 + \frac{3}{x-1}\right| - \frac{7/3}{(x-1)} + c}$$

(sjekket i geogebra...)

$$\int_2^5 \frac{3x}{x^2-3}\,dx = \int_{1=u(2)}^{22} \frac{3/2\,du}{u}$$

$u = x^2-3$

$u' = 2x$

$\frac{3}{2}du = 3x\,dx$

$$= \frac{3}{2}\ln|u|\Big|_1^{22} = \underline{\frac{3}{2}\ln(22)}$$

$$\frac{3x}{x^2-3} = \frac{3x}{(x-\sqrt{3})(x+\sqrt{3})} = \frac{A}{x-\sqrt{3}} + \frac{B}{x+\sqrt{3}} \ldots$$

$$= \frac{3}{2}\left(\frac{1}{x+\sqrt{3}} + \frac{1}{x-\sqrt{3}}\right)$$

$F'(u) = f(u)$

$$\int_a^b u' f(u)\,dx = F u(x)\Big|_a^b$$

$$= F(u(b)) - F(u(a))$$

$$= \int_{u(a)}^{u(b)} f(u)\,du$$

$\frac{d}{dx} F(u(x))$

$= u'(x)\, f(u)$

$$\int \frac{1}{x^4-16}\,dx = \int \frac{1}{(x^2+4)(x^2-4)}\,dx$$

$$= \int \frac{1}{(x^2+4)(x-2)(x+2)}\,dx$$

$$\frac{1}{8}\left(\frac{1}{x^2-4} - \frac{1}{x^2+4}\right) = \frac{1}{x^4-16}$$

$$\frac{1}{x^2-4} = \frac{1}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right) \qquad \text{gir}$$

$$\int \frac{1}{x^4-16}\,dx = \frac{1}{8}\int \frac{1}{x^2-4} - \frac{1}{x^2+4}\,dx$$

$$= \frac{1}{8\cdot 4}\int \frac{1}{x-2} - \frac{1}{x+2}\,dx - \underbrace{\frac{1}{8}\int \frac{1}{x^2+4}\,dx}_{\frac{1}{8}\cdot\frac{1}{4}\int \frac{1}{(x/2)^2+1}\,dx}$$

$$= \frac{1}{32}\ln\left|\frac{x-2}{x+2}\right| - \frac{1}{32}\cdot 2\arctan\left(\frac{x}{2}\right) + C$$