25.03
2022

16.7 Diff likninger.

Newtons lov for varmeoverføring.

T_{ute} (reservoar)

$\boxed{T(t)}$

$$T'(t) = -k\left(T(t) - T_{ute}\right)$$

$k > 0$

$T(0) = T_0$

$$\int \frac{T'(t)}{T(t) - T_{ute}}\,dt = \int -k\,dt$$

separabel

$$\int \frac{dT}{T - T_{ute}} = -kt + c$$

$$e^{\ln|T-T_{ute}|} = e^{-kt+c}$$

$$|T-T_{ute}| = Ke^{-kt}$$

$$T-T_{ute} = Ke^{-kt}$$

$$T(t) = T_{ute} + Ke^{-kt}$$

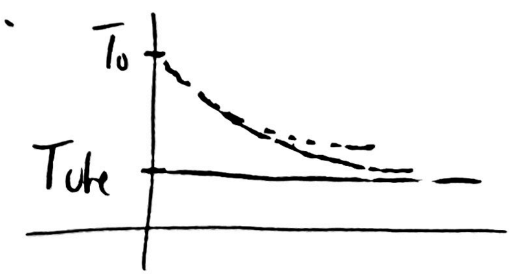
$T(0)=T_0$
initialbetingelse

$$T(0)=T_0 = T_{ute} + K\cdot 1$$

så $K = T_0 - T_{ute}$

$$\underline{T(t) = T_{ute} + (T_0 - T_{ute})e^{-kt}}$$

$T(t) \to T_{ute}$ når $t \to \infty$.



opg

$T'(t) = -\ell(T - 20°C)$

$T(0) = 100°C$ $\quad$ $T(4\,min) = 80°C$.

Hva er $T(t)$ $\quad$ Når er $T = 50°C$?

$\int \frac{T'}{T-20}\,dt = \int -\ell\,dt$ gir $\ln|T-20| = -\ell t + c$

$\underline{T - 20 = K e^{-\ell t}}$

$T(0) = 100$ så $100 - 20 = K e^{0} = K$, $K = 80°C$

$\underline{T(t) = 20 + 80 e^{-\ell t}}$ $\quad$ t enheter minutter.

$T(4) = 80$ gir $80° = 20 + 80 e^{-4\ell}$

$$\frac{80-20}{80} = e^{-4\ell}$$

$$\frac{3}{4} = e^{-4\ell}$$

$$\ln(3/4) = \ln e^{-4\ell} = -4\ell$$

$$\ell = \frac{\ln(3/4)}{-4} = \underline{\frac{\ln(4/3)}{4}} \sim \underline{0{,}072 \text{ min}^{-1}}$$

tiden när $T = 50^\circ$:

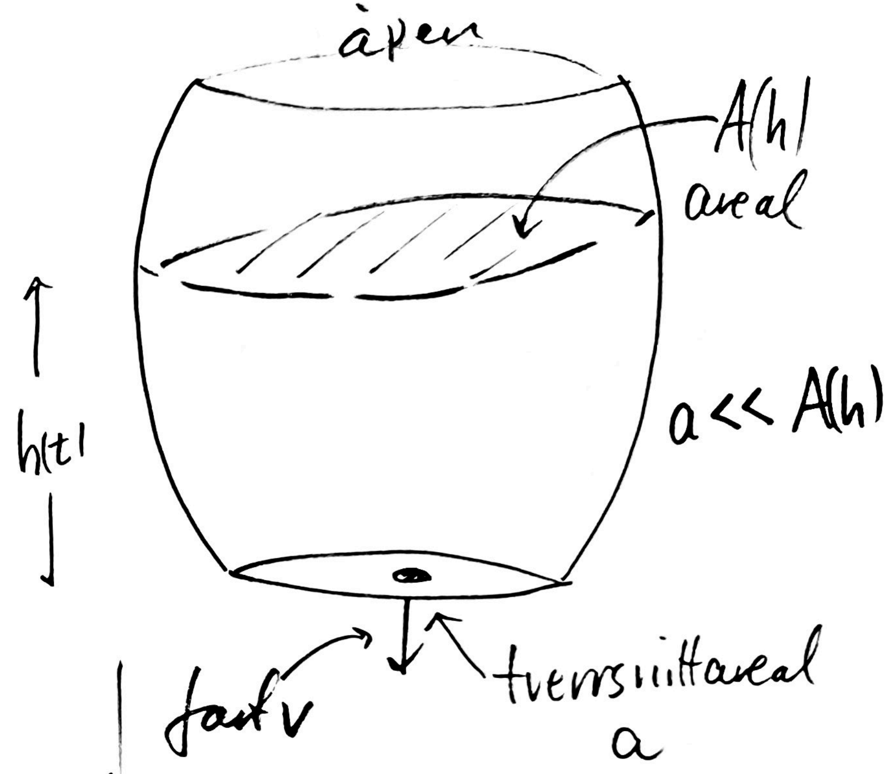
$$50 = 20 + 80\, e^{-\ell t_{50}}$$

$$\frac{3}{8} = e^{-\ell t_{50}}$$

$$+\ln(3/8) = -\ln(8/3) = -\ell\, t_{50}$$

$$t_{50} = \frac{\ln(8/3)}{\ell} = \underline{4\, \frac{\ln(8/3)}{\ln(4/3)}} \sim \underline{13.64 \text{ min}}$$

Torricellis lov



Δt like tidspunkt.

endring i høyde $\Delta h < 0$

endring i volum $\Delta V = -A(h) \cdot \Delta h$

$\Delta V = a \cdot v \, \Delta t$

Bevaring av energi : $\overset{\text{masse}}{\Delta m} = \underset{\uparrow \text{massetetthet}}{\rho} \cdot \Delta V$

$$\frac{1}{2} \Delta m \, v^2 = \Delta m \cdot g \cdot h$$

gir $v = \sqrt{2gh}$

$$-\Delta h \, A(h) = \Delta V = a \cdot v \cdot \Delta t$$

$$-\frac{\Delta h}{\Delta t} = a \cdot v \cdot \frac{1}{A(h)}$$

la $\Delta t \to 0$

$$-h'(t) = \frac{a}{A(h)} \cdot v = \frac{a}{A(h)} \sqrt{2gh}$$

$$\boxed{\text{Torricellis lov} \quad h'(t) = -\frac{a}{A(h)} \sqrt{2gh}}$$ separabel

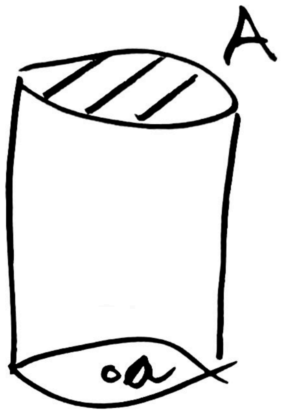
$h(0) = H$

$$\frac{A(h)\, h'}{\sqrt{h}} = -a\sqrt{2g}$$

$$\int_0^t \frac{A(h)}{\sqrt{h}} h' \, dt = \int_0^t -a\sqrt{2g} \, dt = -a\sqrt{2g} \cdot t$$

$$\int_H^{h(t)} \frac{A(h)}{\sqrt{h}} \, dh = -a\sqrt{2g} \cdot t$$

Rett sylindrisk tank



$A(h)$ konstant

$$\int_H^{h(t)} A \overbrace{\frac{1}{\sqrt{h}}}^{h^{-1/2}} dh = -a\sqrt{2g}\,t$$

$$A \cdot \frac{h^{1/2}}{1/2} \Big|_H^{h(t)} = 2A\left(\sqrt{h(t)} - \sqrt{H}\right) = -a\sqrt{2g}\,t$$

$$\sqrt{h(t)} = \sqrt{H} - \frac{a}{2A}\sqrt{2g}\,t$$

$$\underline{h(t) = \left(\sqrt{H} - \frac{a}{2A}\sqrt{2g}\,t\right)^2}$$

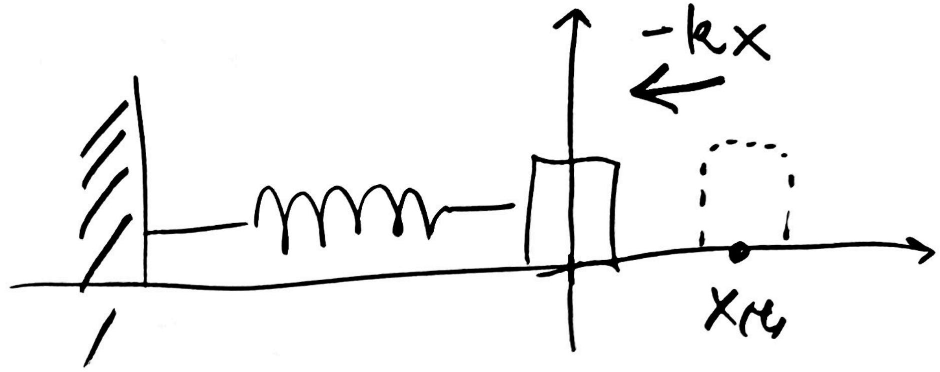
gyldig til tanken tømmes.

$$\sqrt{H} = \frac{a}{2A}\sqrt{2g}\,T$$

Tiden det tar å tømme tanken

$$T = \frac{2A\sqrt{H}}{a\sqrt{2g}}.$$

Harmonisk svingning



Hooks lov

Kraft proporsjonal til forflytting fra jamvektsposisjon

$k > 0$

$$m x'' = -kx$$

$$x'' + \frac{k}{m} x = 0$$

Løsninger

$$A \sin\left(\sqrt{\frac{k}{m}}\, t\right) + B \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

Dempet harmonisk svingning

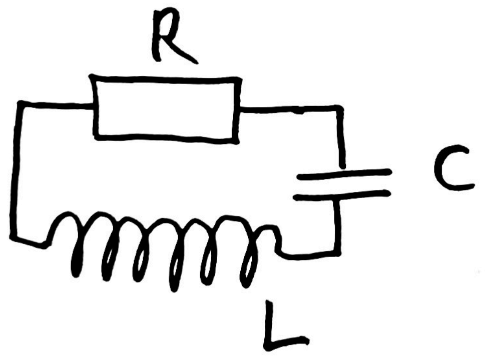
Friksjon proporsjonal til farten.

$$m x'' = -kx - \ell x'$$

$$x'' + \left(\frac{\ell}{m}\right) x' + \frac{k}{m} x = 0$$

Løsning på formen $e^{-\frac{\ell}{m \cdot 2} t} \sin(bt + c)$

$q(t)$ ladning



$$\frac{q}{C} + R q'(t) + L q'' = 0$$

$$q'' + \frac{R}{L} q' + \frac{1}{LC} q = 0$$

samme diff likning som i det mekaniske eksempelet!