18april Luftmotstanden viser seg å være proporsjonal til fart kvadrert : $\ell \cdot V^2$.

ℓ konstant.

x

$x' = V$ fart > 0

$x'' = V' = a$ akselerasjon

Newtons andre lov $m \cdot a = \Sigma$ krefter

$$m \cdot V' = m \cdot g - \ell \cdot V^2.$$

Når legemet faller vil farten stabilisere seg.

$V' = 0$: $m \cdot g - \ell \cdot V^2 = 0$

så stabil fart er: $V_{st} = \sqrt{\frac{m \cdot g}{\ell}}$.

Hvis $V_{st} = 60 m/s$ (~220 km/t) så er $\frac{\ell}{m} = \underline{0{,}0027 \, m^{-1}}$

Vi løser differensiallikningen

29 april

$$mV' = mg - \ell \cdot V^2$$

$$V' = g - \left(\frac{\ell}{m}\right) \cdot V^2$$

$$\frac{V'}{g - \frac{\ell}{m} \cdot V^2} = 1$$

alternativt: $\frac{\frac{m}{\ell} \cdot V'}{\frac{mg}{\ell} - V^2} = 1$

Delbrøksoppspalting : $\frac{m}{\ell} V' \frac{1}{\left(\sqrt{\frac{mg}{\ell}} - V\right)\left(\sqrt{\frac{mg}{\ell}} + V\right)} = 1$

$$\frac{m}{l} v' \left(\frac{1}{\left(\sqrt{\frac{mg}{l}} - v\right)} + \frac{1}{\sqrt{\frac{mg}{l}} + v} \right) \frac{1}{2\sqrt{\frac{mg}{l}}} = 1$$

integrerer

$$\frac{m}{l}\,\frac{1}{2\sqrt{\frac{mg}{l}}} \int \frac{1}{\sqrt{\frac{mg}{l}} - v} + \frac{1}{\sqrt{\frac{mg}{l}} + v}\, dv = \int 1\, dt$$

$$= \frac{1}{2\sqrt{\frac{m\cdot g}{l}\cdot\frac{l}{m}}} \left[-\ln\left|\sqrt{\frac{mg}{l}} - v\right| + \ln\left|\sqrt{\frac{mg}{l}} + v\right| \right] = t + c$$

$$= \frac{1}{\underbrace{2\sqrt{\frac{m\cdot g}{l}\cdot\frac{l^2}{m^2}}}_{2\sqrt{\frac{gl}{m}}}} \ln\left| \frac{\sqrt{\frac{mg}{l}} + v}{\sqrt{\frac{mg}{l}} - v} \right| = t + c$$

$$\left| \frac{\sqrt{\frac{mg}{l}} + v}{\sqrt{\frac{mg}{l}} - v} \right| = e^{2\sqrt{\frac{gl}{m}}\cdot t + c}$$

$$= e^{c} \cdot e^{2\sqrt{\frac{gl}{m}}\, t}$$

Lineær likning i variabel v

$$\frac{\sqrt{\frac{mg}{l}} + v}{\sqrt{\frac{mg}{l}} - v} = K \cdot e^{2\sqrt{\frac{gl}{m}}\cdot t}$$

$$\sqrt{\frac{mg}{l}} + v = \left(\sqrt{\frac{mg}{l}} - v\right) K e^{2\sqrt{\frac{gl}{m}}\cdot t}$$

$$v\left(1 + K e^{2\sqrt{\frac{gl}{m}}\, t}\right) = \sqrt{\frac{mg}{l}} \left(K e^{2\sqrt{\frac{gl}{m}}\cdot t} - 1 \right)$$

Så $$\underline{v(t) = \sqrt{\frac{mg}{l}} \cdot \frac{K e^{2\sqrt{\frac{gl}{m}}\cdot t} - 1}{K e^{2\sqrt{\frac{gl}{m}}\cdot t} + 1}}$$

Randbetingelsen $v(0) = 0$ gir : $K = 1$.

Da er løsningen

$$v(t) = \sqrt{\frac{mg}{l}} \cdot \frac{e^{2\sqrt{\frac{gl}{m}}t} - 1}{e^{2\sqrt{\frac{gl}{m}}t} + 1} = \underline{\sqrt{\frac{mg}{l}}\left(1 - \frac{2}{e^{2\sqrt{\frac{gl}{m}}\cdot t} + 1}\right)}$$

Hvor lang tid tar det før vi oppnår 90% av stabil fart?

Da må $1 - \frac{2}{e^{2\sqrt{\frac{gl}{m}}t} + 1} = \frac{9}{10} = 90\%$

Så $\frac{2}{e^{2\sqrt{\frac{gl}{m}}} + 1} = \frac{1}{10}$

så $e^{2\sqrt{\frac{gl}{m}}\cdot t} = 20 - 1 = 19.$

$$t_{90\%\ \text{fart}} = \frac{\ln 19}{2\sqrt{\frac{g\cdot l}{m}}}$$

$\frac{l}{m}$ som tidligere

$$\approx \frac{2.94}{2\sqrt{9.8\,m/s^2 \cdot 0.027\,m^{-1}}} \approx \underline{9\ \text{sekund}}$$

Posisjonen $s(t)$ er gitt ved $s'(t) = v(t)$

$$s(t) = \int v(t)\,dt$$
$$= \sqrt{\frac{mg}{l}}\left(t + \sqrt{\frac{m}{gl}}\ln\left(1 + e^{-2\sqrt{\frac{gl}{m}}t}\right)\right) + C$$

Randbetingelsen $s(0) = H$ gir

$$s(t) = \sqrt{\frac{mg}{l}}\cdot t + \frac{m}{l}\left(\underbrace{\ln\left(1 + e^{-2\sqrt{\frac{gl}{m}}t}\right) - \ln 2}_{\text{negativ}}\right) + H$$