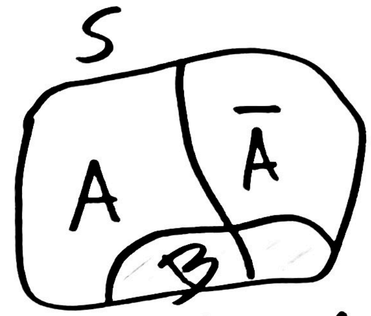
19 april 2022

19.6 Total sannsynlighet



$A \cap \bar{A} = \emptyset \quad A \cup \bar{A} = S$

$B = B \cap A \cup B \cap \bar{A}$

disjunkt.

$P(B) = P(B \cap A) + P(B \cap \bar{A})$

Betinget sannsynlighet

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

så $\underline{P(A \cap B) = P(A) \cdot P(B|A)}$

Tilsvarende for $\bar{A}$.

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Tilsvarende for $\bar{A}$.

opg. $P(B) = 0.4$

$P(A) = 0.6$

$P(B|A) = 0.3$

$P(\bar{A}) = 1 - P(A) = 0.4$

Hva er $P(B|\bar{A})$?

$P(B) = P(A) \cdot P(B|A) + P(\bar{A}) \cdot \underline{P(B|\bar{A})}$ ukjent

$0.4 = 0.6 \cdot 0.3 + 0.4 \cdot P(B|\bar{A})$

så $P(B|\bar{A}) = \dfrac{0.4 - 0.18}{0.4} = \dfrac{0.22}{2/5}$

$= \dfrac{5}{2}(0.22) = 5 \cdot 0.11 = 0.55$

$= \underline{55\%}$

$$P(B) = P(B \cap A) + P(B \cap \bar{A})$$

$$\underline{P(B) = P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A})} \; .$$

Eksempel

S populasjon

A vaksinert

$\bar{A}$ uvaksinerte

B blir syke

$P(B|\bar{A}) = 20\%$

$P(A) = 95\%$

$P(B|A) = 1\%$

$P(\bar{A}) = 1 - P(A) = 5\%$

Hva er $P(B)$?

$$P(B) = P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A})$$

$$0.95 \cdot 0.01 + 0.05 \cdot 0.2$$

$$= 0.0095 + 0.01 = \underline{1.95\%}$$

opg 19.166 $\quad L$ per lyver $\quad \bar{L}$ per snakker sant.

Per kaster en terning. Vi spør om det ble en sekser.

a) Hva er sannsynligheten for at han svarer ja?

sekser A $\quad$ $\bar{A}$ ikkje sekser. $\quad$ $P(L)=0.75$

$P(A) = \frac{1}{6}$ $\quad$ $P(\bar{A}) = \frac{5}{6}$. $\quad$ $P(\bar{L})=0.25$

svarer ja hvis $(A \cap \bar{L}) \cup (\bar{A} \cap L)$

$$P(A \cap \bar{L}) + P(\bar{A} \cap L)$$

$$= P(A) \cdot \underbrace{P(\bar{L} | A)}_{P(\bar{L})} + P(\bar{A}) \cdot \underbrace{P(L | \bar{A})}_{P(L)}$$

$$= \frac{1}{6} \cdot 0.25 + \frac{5}{6}(0.75) = \frac{1}{6 \cdot 4} + \frac{5}{6} \cdot \frac{3}{4}$$

$$= \frac{16}{24} = \underline{\underline{\frac{2}{3}}}$$

b) Hvis svaret er ja, hva er sannsynligheten at han fikk en sekser?

$$P(A \mid A \cap \bar{L} \cup \bar{A} \cap L) = \frac{P(A \cap \bar{L})}{P(A \cap \bar{L} \cup \bar{A} \cap L)} = \frac{1/24}{16/24} = \underline{\frac{1}{16}}$$

ex 2021 mai

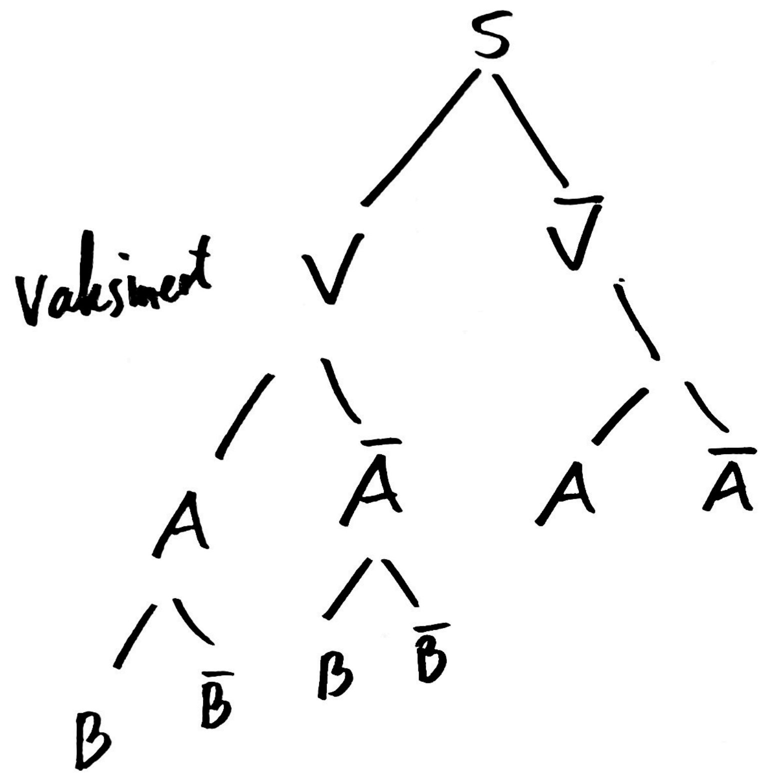
8 Kostnad uten vaksine $1000 \cdot 2.3\% \cdot 10^6 = \underline{23 \cdot 10^6}$

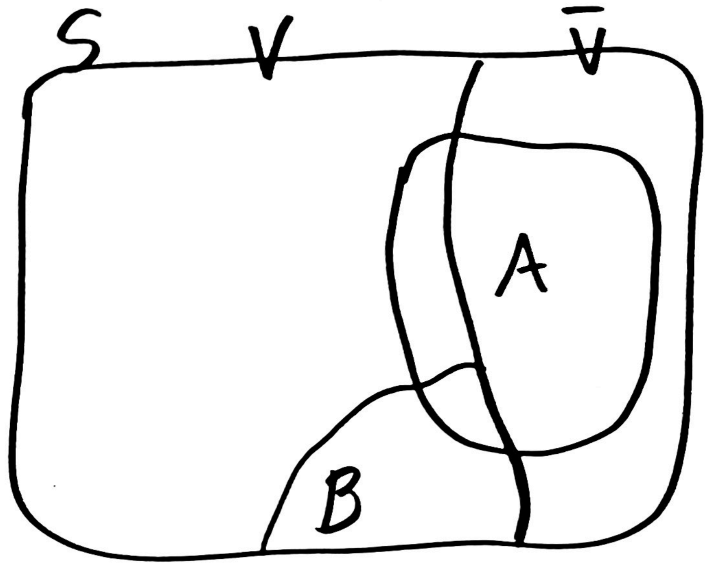
a) Kostnad med vaksine $1000 \cdot 2300 + 2.3 \cdot 10^6$

10% blir fremdeles syke

$= \underline{4.6 \cdot 10^6}$

0.1% av de som vaksineres utvikler sykdom B.





$P(V) = 0.75 \qquad P(\bar{V}) = 0.25$

Hva er $P(A) = P\left((A \cap V) \cup (A \cap \bar{V})\right) = P(A \cap V) + P(A \cap \bar{V})$

$= P(V)\,P(A|V) + P(\bar{V}) \cdot P(A|\bar{V})$.

$= 0.75 \cdot 0.0023 + 0.25 \quad 0.023$

$= (0.075 + 0.25) \cdot 0.0\,23$

$= (0.325 \cdot 0.023)$

$= 0.007475$

$\sim \underline{0.75\%}$

Når populasjonen er 1000. Forventer vi 7,5 personer bliv syke

d) $P(V|A) = \dfrac{P(V \cap A)}{P(A)}$

$$= \frac{0.075 \cdot \cancel{0.023}}{0.325 \cdot \cancel{0.023}} = \frac{75}{325} \sim 0.23$$

$$= \underline{23\%}$$

19.164 Trim : T $\qquad P(T) = \frac{3}{5} = 0.6$

Mye fravær M. $\qquad P(\overline{T}) = 0.4$

$P(M|T) = \frac{1}{8} \qquad P(M|\overline{T}) = \frac{1}{4}$

$$P(M) = P(T) \cdot P(M|T) + P(\overline{T}) \cdot P(M|\overline{T})$$

$$0.6 \cdot \frac{1}{8} + 0.4 \cdot \frac{1}{4}$$

$$= 0.1\left(\frac{6}{8} + 1\right) = 0.175$$

$$= \underline{17.5\%}$$

19.163

E Bilen er ≥ 10 år $\qquad P(E) = 10\%$

$Y = \overline{E}$ Bilen er < 10 år $\qquad P(Y) = 1 - 10\% = 90\%$

M bilen får mangellapp.

a) $P(E \cap M)$ "eldre og får mangellapp".

$$= \underbrace{P(M|E)}_{\frac{1}{4}} \cdot \underbrace{P(E)}_{10\%} = 2.5\%$$

b) $$P(Y \cap M) = \underbrace{P(M|Y)}_{\frac{1}{9}} \cdot \underbrace{P(Y)}_{0.9} = 0.1 = 10\%$$

c) $$P(M) = P(E \cap M) + P(Y \cap M) = 2.5\% + 10\% = \underline{12.5\%}$$

$$P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{2.5\%}{12.5\%} = \frac{1}{5} = \underline{20\%}$$