25.04. 2022

Deriver – $\ln(e^x \cdot 2x^7\sqrt{x})$

– $e^{-2x}\cos(3x+1)$

Regn frem til 11:30

Løs likning-ene – $\frac{1}{x^2-3} = 1 - \frac{4}{5}$

– $\cos(2x) = \frac{\sqrt{3}}{2}$ for $0 \leq x < \pi$

– $e^{2x} - 3e^x - 4 = 0$

Gitt tre punkt $A(1,2,4)$, $B(-2,3,3)$ og $C(0.0.0)$ (origo).

- Finn vinkelen $\angle ABC$
- Finn arealet til trekant ABC.

- Forenkler uttrykket før vi deriverer.

$$\ln(e^x \cdot 2\underbrace{x^7\sqrt{x}}_{x^7 x^{1/2}}) = \underbrace{\ln(e^x)}_{x} + \ln 2 + \underbrace{\ln(x^{7,5})}_{7.5 \cdot \ln x}, \quad x>0$$

$$\left(\ln(e^x \cdot 2x^7\sqrt{x})\right)' = 1 + 0 + 7.5 \cdot \frac{1}{x}$$

$$= \underline{1 + \frac{7.5}{x}} \qquad x>0$$

$$\left(e^{-2x}\cos(3x+1)\right)' = (e^{-2x})'\cos(3x+1) + (e^{-2x})(\cos(3x+1))'$$

$$= e^{-2x}\underbrace{(-2x)'}_{-2}\cos(3x+1) + e^{-2x}(-\sin(3x+1)\cdot\underbrace{(3x+1)'}_{3})$$

$$= \underline{e^{-2x}(-2\cos(3x+1) - 3\sin(3x+1))}$$

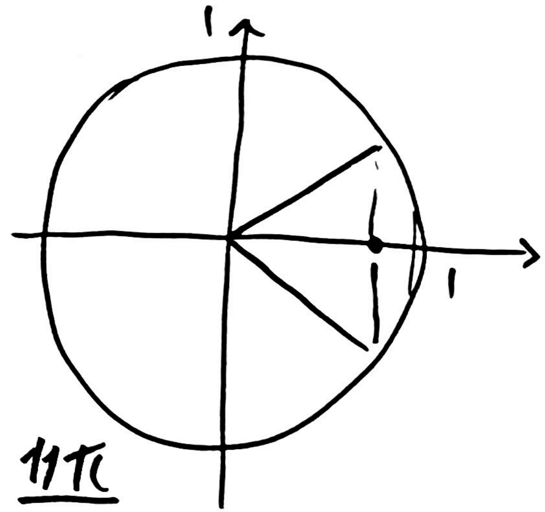
$$\frac{+1}{x^2-3} = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\Leftrightarrow \quad x^2 - 3 = 5 \quad \Leftrightarrow \quad x^2 - 8 = 0$$

$$\Leftrightarrow \quad x^2 = 8 \quad \Leftrightarrow \quad x = \pm\sqrt{8} = \pm\sqrt{2 \cdot 4} = \underline{\pm 2\sqrt{2}}$$

$$\cos(\underbrace{2x}_{u}) = \sqrt{3}/2 \qquad 0 \leq x < \pi$$

$$0 \leq u \leq 2\pi$$



$$u = \pm\arccos\left(\frac{\sqrt{3}}{2}\right) + 2\pi \cdot n$$

$$u = \pm \pi/6 + 2\pi \cdot n \qquad n \text{ heltall.}$$

mellom 0 og 2π er $u = \pi/6$ og $\frac{11\pi}{6}$

$x = \frac{u}{2}$ Løsningene er $\underline{x = \frac{\pi}{12} \text{ og } \frac{11\pi}{12}}$.

$e^{2x} - 3e^{x} - 4 = 0$ benytter $e^{2x} = (e^{x})^2$

$(e^{x})^2 - 3e^{x} - 4 = 0$ 2.gradslikning i e^{x}

$u = e^{x}$

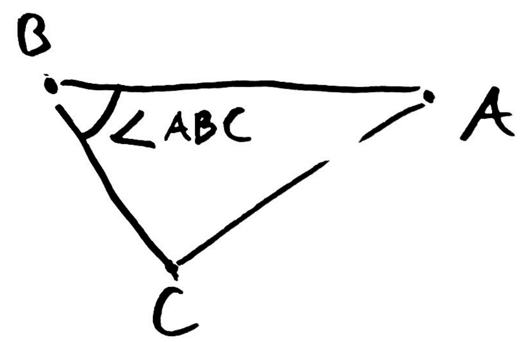
$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$ løsningene er:

$u = 4$ og $u = -1$

$e^{x} = 4$ $e^{x} = -1$ (ingen reell løsning)

Løsningen er $\underline{x = \ln(4)}$



$$\vec{BA} = \vec{OA} - \vec{OB}$$
$$= [1,2,4] - [-2,3,3]$$
$$\vec{BA} = [3,-1,1]$$

$$\vec{BC} = \vec{BO} = -\vec{OB} = -[-2,3,3] = [2,-3,-3]$$

$$\vec{BA} \cdot \vec{BC} = \cos(\angle ABC) \cdot |\vec{BA}| \cdot |\vec{BC}|$$
$$\cos(\angle ABC) = \frac{[3,-1,1] \cdot [2,-3,-3]}{|[3,-1,-1]| \cdot |[2,-3,-3]|}$$

$$= \frac{6+3-3}{\sqrt{9+1+1}\,\sqrt{4+9+9}} = \frac{6}{\sqrt{11}\,\sqrt{22}} = \frac{6}{\sqrt{11}\,\sqrt{2\cdot 11}}$$

$$= \frac{6}{11\cdot\sqrt{2}} = \underline{\frac{3\sqrt{2}}{11}}$$

gir $\angle ABC = \arccos\left(\frac{3\sqrt{2}}{11}\right)$

$\underline{\sim 67.3^\circ}$

Arealet $\frac{1}{2} |\vec{BA}| \cdot |\vec{BC}| \cdot \sin(\angle ABC)$ (Sinussetningen)

$$= \frac{11 \cdot \sqrt{2}}{2} \sin(\angle ABC) \sim 7.176.$$

Alternativt: $A = \frac{1}{2} |\vec{BA} \times \vec{BC}|$

$$\vec{BA} \times \vec{BC} = \begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ 2 & -3 & -3 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ -3 & -3 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix} \vec{k}$$

$$= (3-(-3))\vec{i} - (-9-2)\vec{j} + (-9+2)\vec{k}$$

$$= 6\vec{i} + 11\vec{j} - 7\vec{k} = [6, 11, -7]$$

Arealet $\triangle ABC$ er $A = \frac{1}{2} |[6, 11, -7]|$

$$= \frac{1}{2}\sqrt{36 + 121 + 49} = \underline{\underline{\frac{1}{2}\sqrt{206}}}$$