27.04
2022

odde og jevne funksjoner.

$f(x)$ symmetrisk definisjons-
mengde D_f
$(x \in D_f \Leftrightarrow -x \in D_f)$

$f(x)$ jevn $\quad f(x) = f(-x)$

$f(x)$ odde $\quad -f(x) = f(-x)$

eks.

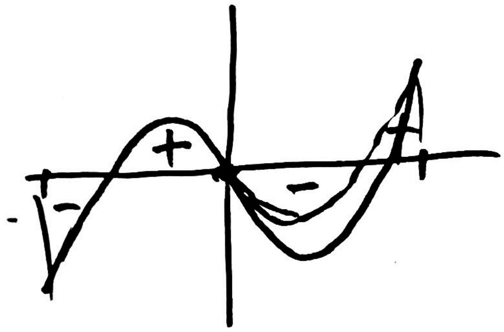
jevne	odde
n jevnt heltall $\quad x^n$	n odde heltall $\quad x^n$
$\cos(x)$	$\sin(x)$
$\lvert x \rvert$	

$$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{jevn}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{odde}}$$

$f(x)$ jevn $\Rightarrow$ $f'(x)$ odde

$f(x)$ odde $\Rightarrow$ $f'(x)$ jevn.

$\int_{-3}^{3} f(x)\,dx = 0$ hvis $f(x)$ er odde

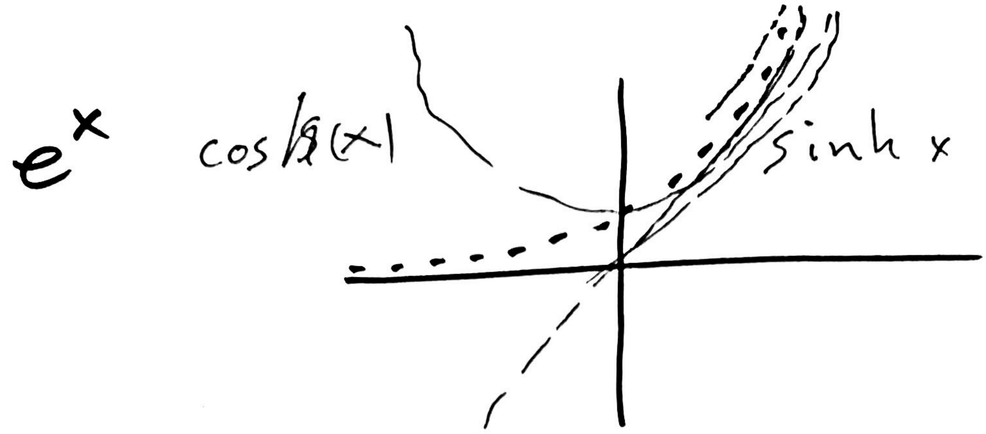


$\int_{-3}^{3} \sin^3(x)\,dx = 0$

siden $\sin^3(x)$ er odde

og vi integrerer over $[-3,3]$.

$$(\sin(-x))^3 = (-\sin(x))^3 = -(\sin(x))^3 \quad \text{odde.}$$



$$e^x = \underbrace{\frac{e^x + e^{-x}}{2}}_{\cosh(x)} + \underbrace{\frac{e^x - e^{-x}}{2}}_{\sinh(x)}$$

$\cosh(x)$ jevn $\sinh(x)$ odde

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$(\cosh(x))' = \frac{1}{2}\left((e^x)' + (e^{-x})'\right)$$
$$= \frac{1}{2}(e^x - e^{-x})$$

$$\boxed{\begin{aligned} (\cosh(x))' &= \sinh(x) \\ (\sinh x)' &= \cosh(x) \end{aligned}}$$

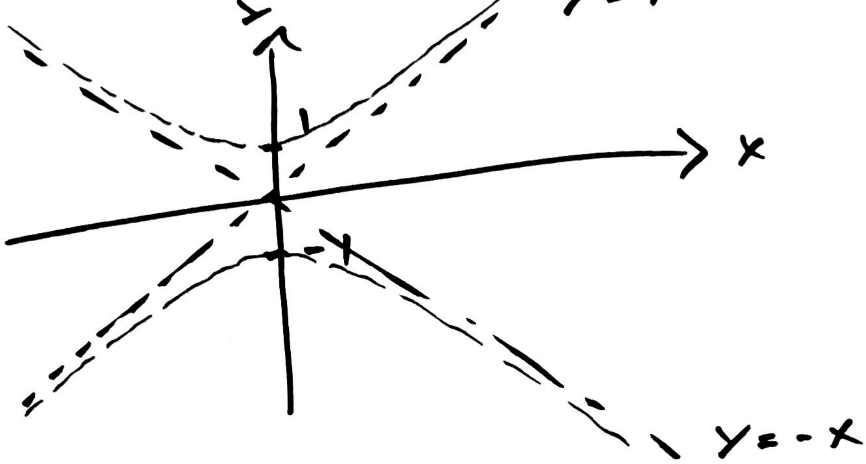
benytter $e^x \cdot e^{-x} = e^{x-x} = e^0 = 1$

$$\cosh^2 x - \sinh^2 x$$
$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$
$$= \frac{1}{4}\left[(e^{2x} + 2\cdot 1 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})\right]$$
$$= 1$$

$$\cosh^2 x - \sinh^2 x = 1$$

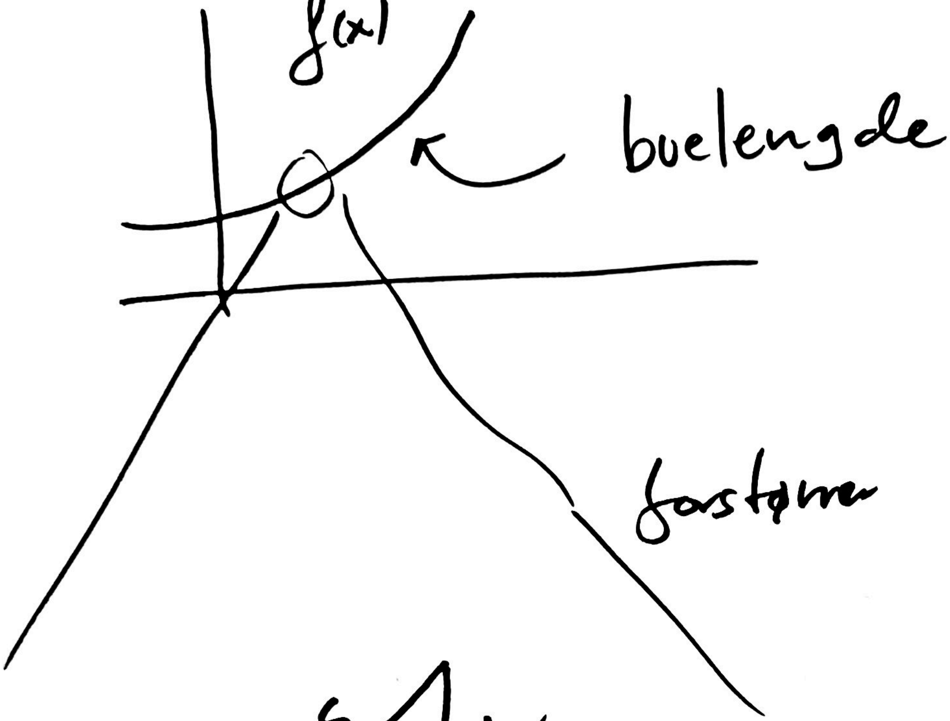
Grafen til $y^2 - x^2 = 1$

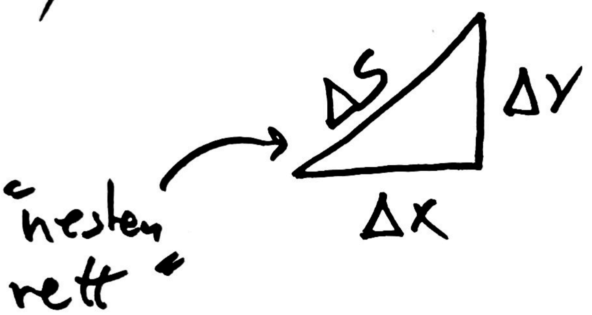
kalles en hyperbel



$(\sinh(t), \cosh(t))$ parametriserer øvre del av hyperbelen

$(\sinh(t), -\cosh(t)$ — " — nedre del av hyperbelen





$$(\Delta x)^2 + (\Delta y)^2 = (\Delta s)^2$$

$$1 + \left(\frac{\Delta y}{\Delta x}\right)^2 = \left(\frac{\Delta s}{\Delta x}\right)^2$$

Lar $\Delta x \to 0$

$$1 + (y'(x))^2 = \left(\frac{ds}{dx}\right)^2$$

$$\boxed{\frac{ds}{dx} = \sqrt{1+(f'(x))^2}}$$

Buelengde fra $x=a$ til $x=b$ er lik

$$s = \int_a^b \sqrt{1+(f'(x))^2}\, dx$$

Eksempel $f(x) = k \cdot x^{3/2}$ $x \geq 0$

$$f'(x) = \frac{3}{2} k \sqrt{x}$$

$$1 + (f'(x))^2 = \sqrt{1 + \left(\frac{3}{2}k\right)^2 x}$$

Buelengde $\int_a^b \sqrt{1 + \frac{9}{4}k^2 x}\, dx$

$$u(x) = 1 + \frac{9}{4}k^2 x$$

$$u' = \frac{9}{4}k^2, \quad du = \frac{9}{4}k^2\, dx$$

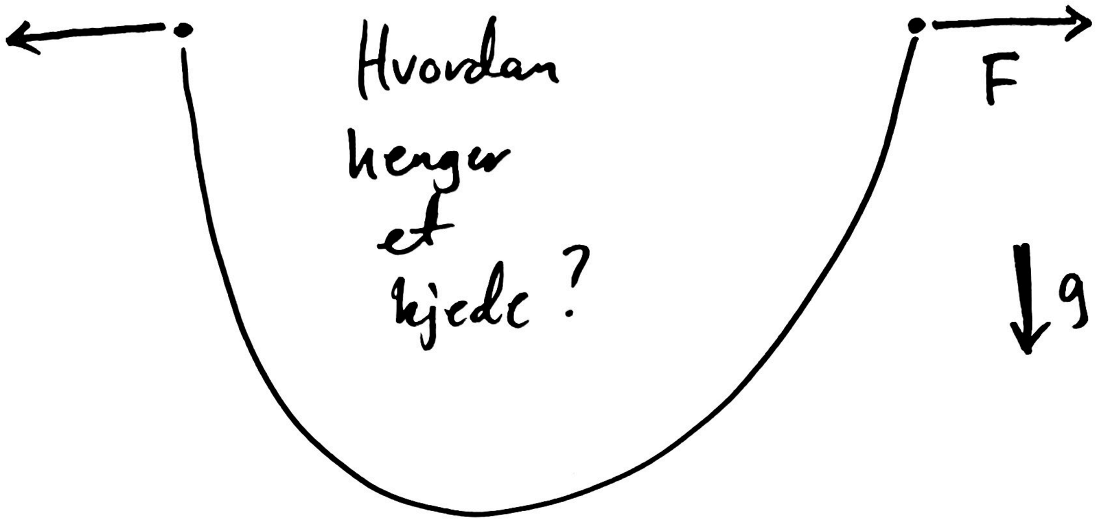
$$\int \sqrt{u(x)}\, dx = \int \underbrace{\sqrt{u}}_{u^{1/2}} \frac{4}{9}\frac{1}{k^2}\, du$$

$$= \frac{u^{3/2}}{3/2} \cdot \frac{4}{9}\frac{1}{k^2} + C$$

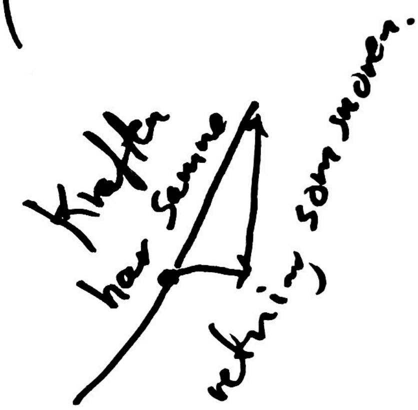
$$= \frac{1}{k^2}\frac{8}{27}\left(1 + \left(\frac{3}{2}k\right)^2 x\right)^{3/2} + C$$

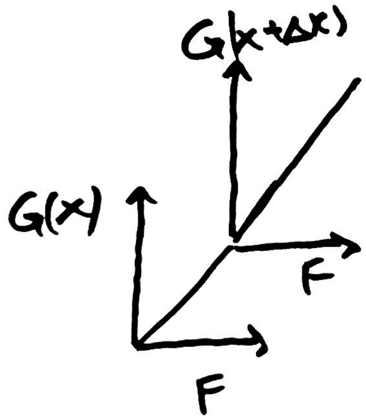
Så $\int_a^b \sqrt{1 + \frac{9}{4}k^2 x}\, dx = \frac{8}{27k^2}\left(1 + \left(\frac{3}{2}k\right)^2 x\right)^{3/2}\Big|_a^b$

$$S = \frac{8}{27k^2}\left[\left(1 + \left(\frac{3}{2}k\right)^2 b\right)^{3/2} - \left(1 + \left(\frac{3}{2}k\right)^2 a\right)^{3/2}\right].$$



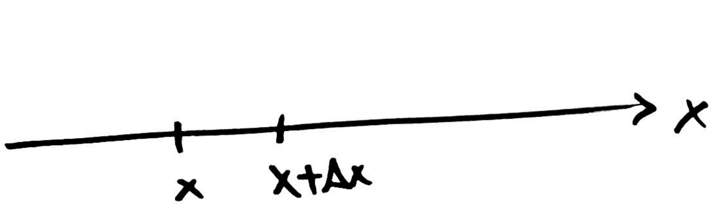
- Henger helt fritt
- Jevn massetetthet ρ (kg/m)





$$\frac{G(x)}{F} = y'(x)$$

$$\frac{G(x+\Delta x)}{F} = y'(x+\Delta x)$$



$$\Delta G = G(x+\Delta x) - G(x)$$

$= g \cdot$ (massen til hjedet mellom x og $x+\Delta x$)

gravitasjons-konstan

$$= g \cdot \rho \cdot (\text{buelengden mellom } x \text{ og } x+\Delta x)$$

$$= g \cdot \rho \sqrt{1+(y'(x))^2} \cdot \Delta x$$

$$\frac{1}{F}\frac{\Delta G}{\Delta x} = \frac{y'(x+\Delta x) - y'(x)}{\Delta x}$$

La $\Delta x \to 0$

$$\boxed{\frac{1}{F} \cdot g \cdot \rho \sqrt{1+(y')^2} = y''(x)}$$

ikke lineær diff likning.

av orden 2 i y

(men av orden 1 i $y'(x)$)

La $y' = f$, $\underline{\frac{\rho g}{F}\sqrt{1+f^2} = f'}$

Løsningen er $f(x) = \sinh\left(\frac{\rho g}{F}(x+c)\right)$

$$f'(x) = \frac{\rho g}{F}\cosh\left(\frac{\rho g}{F}(x+c)\right)$$

$$y' = \sinh(k(x+c_1))$$

hvor $k = \frac{\rho g}{F}$

$$y = \frac{1}{k}\cosh(k(x+c_1)) + c_2$$

c_1 horisontal forskyvning

c_2 vertikal forskyvning.