29.04.2022

OPPGAVER

Løs likningene a) $\frac{1}{x+2} + \frac{1}{x-1} = 0.7$

b) $e^{\sin(x+1)} = 2$ $\quad x \in (-\pi, \pi)$

c) $\arccos(y) = 1$ (vinkelenhet er radianer)

Finn integralene a) $\int (2x+1)e^{-x}\,dx$

b) $\int_{-2}^{2} 4 + 3x + x^2 - 5x^3\,dx$

Finn summen av den geometriske rekken

$$\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \dots + \frac{1}{256}$$

Finn en likning for planet vinkelrett på vektoren $[-2, 3, 1]$ som inneholder punktet $(2, 1, -1)$.

1 a) $\frac{1}{x+2} + \frac{1}{x-1} = 0.7 \quad | \; (x+2)(x-1)$

$$(x-1) + (x+2) = 0.7(x+2)(x-1)$$

$$(2x+1) = 0.7(x^2+x-2) = \frac{7}{10}(x^2+x-2)$$

$$20x+10 = 7x^2+7x-14$$

$$7x^2-13x-24=0$$

benytter 2.gradsformelen

$$x = \frac{13 \pm \sqrt{13^2-7\cdot 4(-24)}}{2\cdot 7} = \frac{13 \pm \sqrt{169+7\cdot 96}}{2\cdot 7}$$

$$x = \frac{13 \pm \sqrt{800+69-28}}{2\cdot 7} = \frac{13 \pm \sqrt{841}}{2\cdot 7}$$

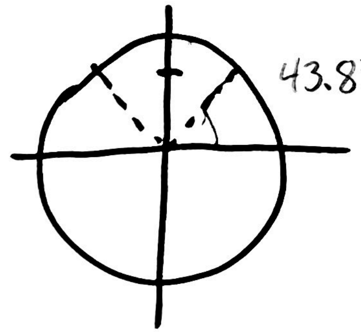
(Vi har: $30^2=900$ $\quad (29)^2=(30-1)^2 = 30^2-2\cdot 30+1 = 841$)

$$x = \frac{13 \pm 29}{2\cdot 7}$$

Løsningene er $\underline{x=3}$

og $\underline{x = \frac{-8}{7}}$

1b) $e^{\sin(x+1)} = 2 \qquad x \in (-\pi, \pi)$



43.87...°

$\Leftrightarrow \sin(x+1) = \ln 2 \sim 0.6931\ldots$

$x+1 = \arcsin(\ln 2) + 2\pi \cdot n$

$\quad = \pi - \arcsin(\ln 2) + 2\pi \cdot n$

$x = \underline{\arcsin(\ln 2) - 1} \sim \underline{-0.234}$

$\quad = \underline{\pi - \arcsin(\ln 2) - 1} \sim \underline{1.375}$

c) $\arccos(y) = 1$ (radian)

$y = \underline{\cos(1)} \sim 0.5403$

2 a) $\int (2x+1)e^{-x}\,dx = 2\int xe^{-x}\,dx + \underbrace{\int e^{-x}\,dx}_{-e^{-x}+C}$

$\int \underset{u}{x}\,\underset{v'}{e^{-x}}\,dx = \overset{u}{x}\,\overset{v}{(-e^{-x})} - \int \overset{u'}{1}\cdot\overset{v}{(-e^{-x})}\,dx$ delvis integrasjon

$= -xe^{-x} - e^{-x} + C$

$\int (2x+1)e^{-x}\,dx = 2(-xe^{-x} - e^{-x}) + -e^{-x} + C$

$= \underline{-2xe^{-x} - 3e^{-x} + C}$

b) $\int_{-2}^{2} 4+3x+x^2-5x^3\,dx = \int_{-2}^{2} 4+x^2\,dx + \underbrace{\int_{-2}^{2} 3x-5x^3\,dx}_{0}$

0 siden $3x-5x^3$ er en odde funksjon

$= \left[4x + \frac{x^3}{3}\right]_{-2}^{2} = 4(2-(-2)) + \frac{1}{3}(2^3-(-2)^3)$

$= 4\cdot 4 + \frac{1}{3}\cdot 16 = 16\left(1+\frac{1}{3}\right) = \underline{\frac{64}{3}} = \underline{21+\frac{1}{3}}$

3 $\quad \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \cdots + \frac{1}{256}$ geometrisk rekke

kvotienten k må være $k = \frac{1}{\sqrt{2}}$

$$1 + k + k^2 + \cdots + k^n = \begin{cases} \frac{1-k^{n+1}}{1-k} & k \neq 1 \\ n+1 & k = 1. \end{cases}$$

$$256 = 2^8 = \left((\sqrt{2})^2\right)^8 = (\sqrt{2})^{16}$$

$$\frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2 + \cdots + \left(\frac{1}{\sqrt{2}}\right)^{16}$$

$$= \frac{1}{\sqrt{2}}\left(1 + \left(\frac{1}{\sqrt{2}}\right) + \cdots + \left(\frac{1}{\sqrt{2}}\right)^{15}\right)$$

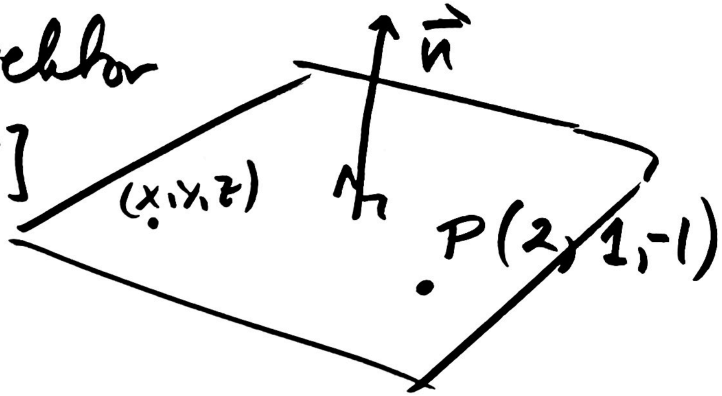
$$= \frac{1}{\sqrt{2}} \frac{1-\left(\frac{1}{\sqrt{2}}\right)^{16}}{\left(1-\frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2}} \frac{1-\frac{1}{256}}{\left(1-\frac{1}{\sqrt{2}}\right)}$$

$$= \underline{\underline{\frac{1-1/256}{\sqrt{2}-1}}} \quad \left(= \left(1-\frac{1}{256}\right)(\sqrt{2}+1) = \frac{255}{256}(\sqrt{2}+1) \right)$$

4.

normal vektor

$\vec{n} = [-2,3,1]$



(x,y,z) er i planet

$$\overrightarrow{P(x,y,z)} \cdot \vec{n} = 0$$

$$[x,y,z] \cdot \vec{n} = \overrightarrow{OP} \cdot \vec{n}$$

$$-2x + 3y + z = [2,1,-1] \cdot [-2,3,1]$$

$$= -4 + 3 - 1 = -2$$

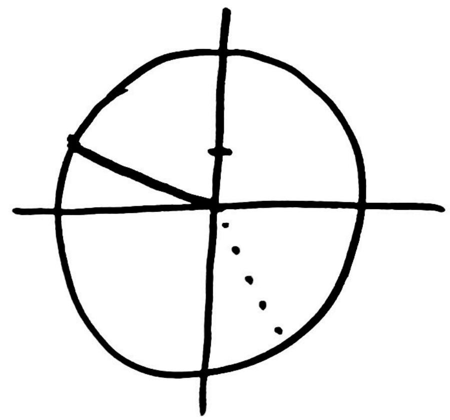
Likningen for planet er $\underline{-2x + 3y + z = -2}$.

(se gjerne eksamen Vår 2019)

Ex2020 Vår

#4 Finn eksakt verdi for $\cos x$, $\sin 2x$ og $\tan x$

når $\sin x = \frac{1}{3}$ og $x \in [90°, 180°]$.



Pytagoras $\sin^2 x + \cos^2 x = 1$

Dobling av vinkel $\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\cos x = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

$$\sin(2x) = 2\sin x \cdot \cos x = 2\cdot\frac{1}{3}\cdot\left(-\frac{2\sqrt{2}}{3}\right) = -\frac{4\sqrt{2}}{9}$$

$$\tan x = \frac{\sin x}{\cos x} = \left(\frac{-2\sqrt{2}/3}{1/3}\right)^{-1} = -\frac{1}{2\sqrt{2}}$$

14.230 d) $\vec{BC} = \vec{OC} - \vec{OB} = [0,2,7] - [4,4,3]$

$= [-4,-2,4]$

$\vec{F}_{BC}$ retning til $\vec{BC}$ og $|\vec{F}_{BC}| = 60\,N$

$$\vec{F}_{BC} = |\vec{F}_{BC}| \cdot \frac{\vec{BC}}{|\vec{BC}|} = 60\,N \frac{[-4,-2,4]}{|[-4,-2,4]|}$$

$$|\vec{BC}| = |[-4,-2,4]| = \sqrt{16+4+16} = \sqrt{36} = 6$$

$$\vec{F}_{BC} = 60N \frac{[-4,-2,4]}{6} = \underline{[-40,-20,40]\,N}$$

e) $\vec{M} = \vec{AB} \times \vec{F}_{BC} = [2,4,1] \times (20[-2,-1,2])$

$$= 20 \begin{vmatrix} i & j & k \\ 2 & 4 & 1 \\ -2 & -1 & 2 \end{vmatrix} = 20[9,-(6),6] = \underline{\underline{60[3,-2,2]}}$$

$u = x^2 \qquad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

* $\int x \sin(x^2)\,dx$

$= \int \sin(u)\frac{1}{2}du = \frac{1}{2}(-\cos(u)) + C$

$= \underline{\frac{-1}{2}\cos(x^2) + C}$

substitusjon

$\int u' f(u)\,dx$

$= \int f(u)\,du.$

* $\int \underbrace{x^2}_{u}\underbrace{e^x}_{v'}dx = u \cdot v - \int u' v\,dx$

$= x^2 e^x - \int 2x e^x dx$

$= x^2 e^x - 2\left[xe^x - \int 1 e^x dx\right]$

$= \underline{x^2 e^x - 2xe^x + 2e^x + C}$

Bruker delvis integrasjon 2ganger