14. okt 2022

(Tilsvarende opg 11 icblig2)

Finn alle x s.a tangentlinjen til

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - 2x + 1$$

i $(x, f(x))$ er parallell til $y = 2x - 3$.

Tangentlinjen i $(x, f(x))$ er parallell til $2x-3$ når stigningstallet til tangentlinjen $f'(x)$ er lik 2

$$f'(x) = 2$$

$$f'(x) = \left(\frac{1}{3}\cdot x^3 + \left(\frac{-3}{2}\right)x^2 - 2x + 1\right)' = \frac{1}{3}\cdot 3x^2 + \left(\frac{-3}{2}\right)2x - 2$$

$$= \underline{x^2 - 3x - 2 = +2}$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

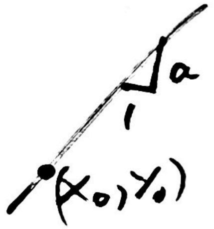
Løsningene er $\underline{x=-1}$ og $\underline{x=4}$.

* Finn tangentlinjen (normallinjen) til $f(x)$ i punktet $(-1, f(-1))$.

$$f(-1) = \frac{(-1)^3}{3} - \frac{3}{2}(-1)^2 - 2(-1) + 1$$

$$= \frac{-1}{3} \underbrace{- \frac{3}{2} + 2}_{\frac{1}{2}} + 1 = \frac{1}{2} - \frac{1}{3} + 1 = \frac{3}{6} - \frac{2}{6} + 1$$

$$= 1 + \frac{1}{6} = \frac{7}{6}$$



$$y = a(x - x_0) + y_0$$

Linje med stigningstall a som går gjennom (x_0, y_0).

Tangentlinjen:

$$y = 2(x - (-1)) + \frac{7}{6}$$

$$= 2x + 2 + \frac{7}{6}$$

$$\underline{y = 2x + 3 + \frac{1}{6}}$$

Normallinjen har stigningstall $\frac{-1}{2}$

$$y = \frac{-1}{2}(x - (-1)) + 1 + \frac{1}{6}$$

$$= \frac{-x}{2} - \frac{1}{2} + 1 + \frac{1}{6} = \frac{-x}{2} + \frac{1}{2} + \frac{1}{6} = \frac{-x}{2} + \frac{3}{6}$$

$$\underline{y = \frac{-x}{2} + \frac{2}{3}}$$

(9) Naturlig def. mengde
Kontinuitet. Deriver hvor mulig.

$\sqrt{x}$ def. for alle $x \geq 0$

$\ell(x) = \sqrt{2x+3}$

def. presis når $2x+3 \geq 0$

$2x \geq -3$ deler med 2:

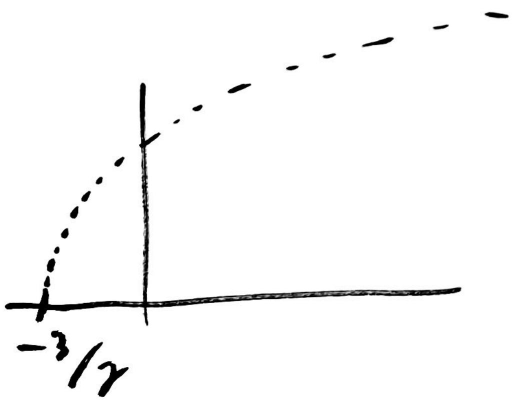
$x \geq -\frac{3}{2}$. $D_\ell = [-\frac{3}{2}, \infty\rangle$

$$\ell'(x) = \left(\sqrt{(2x+3)}\right)' = \frac{1}{2\sqrt{2x+3}} \cdot \underbrace{(2x+3)'}_{2}$$

$$= \frac{1}{\sqrt{2x+3}}$$

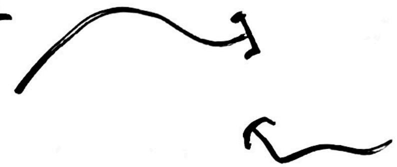
$D_{\ell'} = \langle -\frac{3}{2}, \infty\rangle$

ikkje deriverbar i $x = -\frac{3}{2}$.

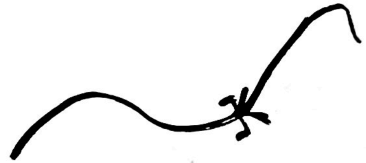


$$f(x) = \begin{cases} g(x) & x \leq 1 \\ h(x) & x > 1 \end{cases}$$

I



II



III

Når er $f(x)$ deriverbar i $x = 1$?

$$f'(1) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

$= f(1)$

$$\lim_{\Delta x \to 0^+} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{h(1+\Delta x) - g(1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0^+} \frac{h(1+\Delta x) - h(1)}{\Delta x} + \frac{h(1) - g(1)}{\Delta x}$$

$$= h'(1) + \lim_{\Delta x \to 0^+} \frac{h(1) - g(1)}{\Delta x}$$

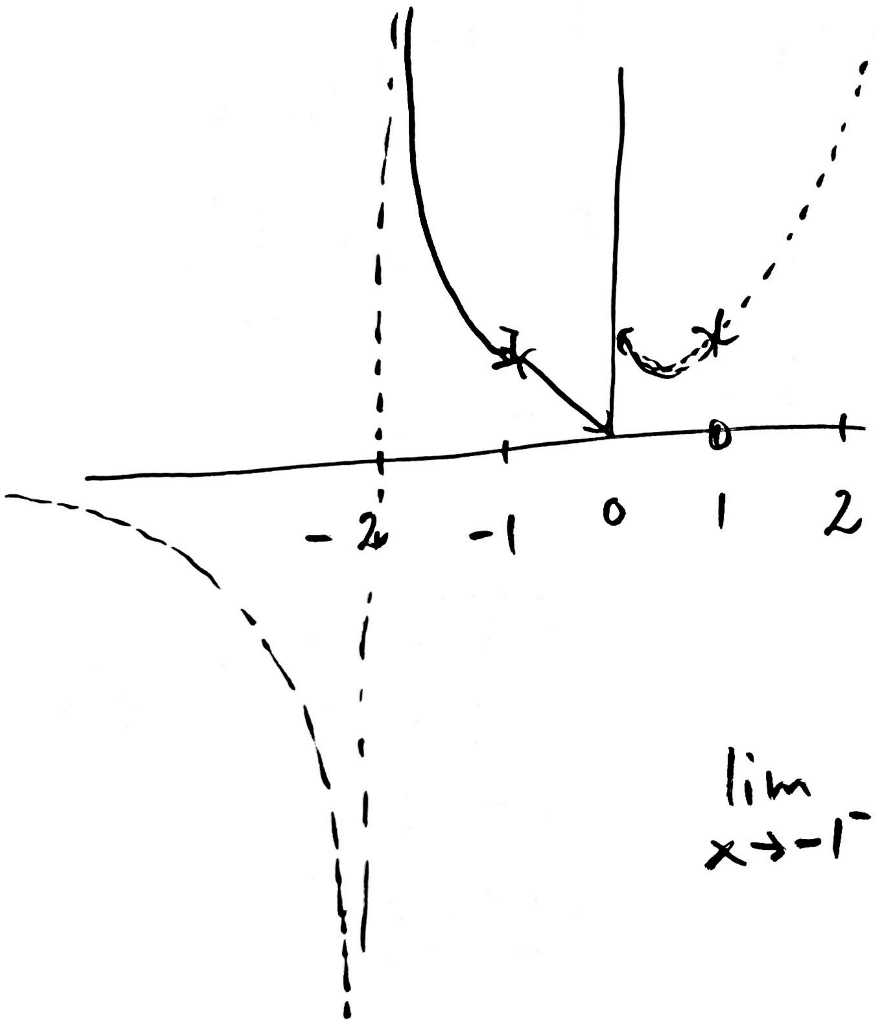
$= h'(1)$ hvis $h(1) = g(1)$.

$$\lim_{\Delta x \to 0^-} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{g(1+\Delta x) - g(1)}{\Delta x} = g'(1)$$

Anta h kan utvides til kont funksjon for $x \geq 1$.

f er deriverbar i $x = 1$ hvis $g(1) = h(1)$ (f er kont. i $x = 1$)

og $g'(1) = h'(1)$.



$$f(x) = \begin{cases} \dfrac{1}{x+2} & x \leq -1,\ x \neq -2 \\ -x & -1 < x < 0 \\ x^2 - x + 1 & 0 \leq x < 1 \\ x^3 - x^2 + 1 & 1 < x \end{cases}$$

$$D_f = \mathbb{R} \setminus \{-2, 1\}.$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{1}{x+2} = \frac{1}{-1+2} = \frac{1}{1} = 1$$

$$= f(-1), \quad \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} -x = 1.$$

Så $\lim\limits_{x \to -1} f(x) = 1 = f(-1)$ så $f(x)$ er kont. i $x = -1$.

$\lim_{x\to 0^-} f(x) = 0 \neq f(0) = 1$ diskont. i $x = 0$.

$\left(\lim_{x\to 1} f(x) = 1 \right.$, f er ikkje def. i $x = 1$)

$f(x)$ er diskont i $x = 0$ (ellers er den kont.)

($x=-1$ lagt til p.g.a argumentet nedenfor)

$$f'(x) = \begin{cases} \dfrac{-1}{(x+2)^2} & x \leqslant -1 \\ -1 & -1 < x < 0 \\ 2x-1 & 0 < x < 1 \\ 3x^2-2x & 1 < x \end{cases}$$

hva med $x=-1$ og $x=0$?

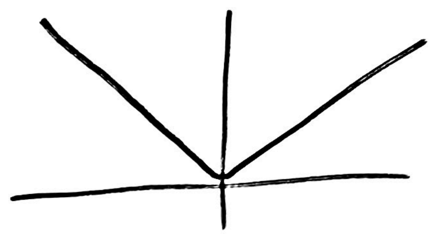
$x=0$ diskont så ikkje deriverbar.

$x=-1$: f er kont. de deriverte fra hver side er:

VS: $\dfrac{-1}{(-1+2)^2} = -1$, HS: -1 samme stigningstall.

så $f(x)$ er deriverbar i $x = -1$.

$$(|x|)' = \frac{|x|}{x}$$



$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$(|x|)' = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

ikkje deriverbar i $x = 0$.

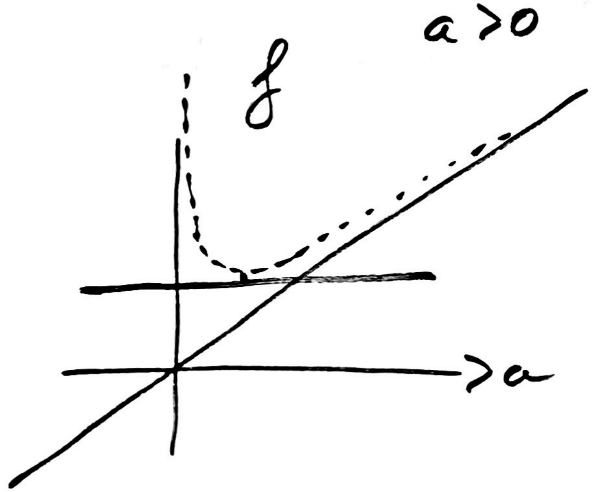
$$\frac{|x|}{x} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

ikkje def. for $x = 0$ (0 over 0).

3 $\quad a + \frac{1}{a} \geq 2$

$\Leftrightarrow \quad a + \frac{1}{a} - 2 \geq 0$

$\Leftrightarrow \quad \frac{a^2 - 2a + 1}{a} \geq 0 \quad \ldots$



Løser opg. ved hjelp av derivasjon:

$$f(a) = a + \frac{1}{a}.$$

minste verdi hvor $f'(a) = 0$

$$f'(a) = 1 + (-1)a^{-2} = 1 - \frac{1}{a^2}.$$

$f'(a) = 0$ når $\quad 1 = \frac{1}{a^2} \quad$ så $\quad a^2 = 1$

Løsningen $(a > 0)$ er $\underline{a = 1}$

Funksjonen er minst når $a = 1$. $\quad f(1) = 1 + \frac{1}{1} = 2.$

$f(a) \geq 2 \quad$ likhet bare for $a = 1$.

(oppg 12)

$$g(x) = \begin{cases} \dfrac{x^2-3x}{2x-3} & x \geq 1 \text{ (ikkje def for } x=3/2) \\ \dfrac{1}{\sqrt{1-x}} & x < 1 \end{cases}$$

$x \geq 1$ pol. div

$$(x^2-3x) : (2x-3) = \underline{\tfrac{1}{2}x - \tfrac{3}{4} + \frac{-9/4}{2x-3}}$$

$$\begin{array}{l} x^2 - \tfrac{3}{2}x \\ \hline \quad -\tfrac{3}{2}x \\ \quad -\tfrac{3}{2}x + \tfrac{9}{4} \\ \hline \quad\quad -\tfrac{9}{4} \end{array}$$

skrå asymptote $\quad y = \frac{1}{2}x - \frac{3}{4}$.

vertikal asymptote $\quad x = 3/2$.

$x < 1$ horisontal asymptote $\quad y = 0$

vertikal — $\quad x = 1$

$$g'(x) = \begin{cases} \frac{1}{2} - \frac{9}{4}\left(\frac{-2}{(2x-3)^2}\right) & x > 1 \\ -\frac{1}{2}\frac{1}{(\sqrt{1-x})^3} \cdot (-1) & x < 1 \end{cases}$$

ikkje kont i $x = 1$

$$\left(\left(\frac{1}{\sqrt{u}}\right)' = \left(u^{-1/2}\right)' = \frac{-1}{2} u^{-\frac{1}{2}-1} = \frac{-1}{2u^{3/2}} \right)$$

$$g'(x) = \begin{cases} \frac{1}{2} + \frac{9}{2(2x-3)^2} & x > 1 \\ \frac{1}{2(\sqrt{1-x})^3} & x < 1 \end{cases}$$

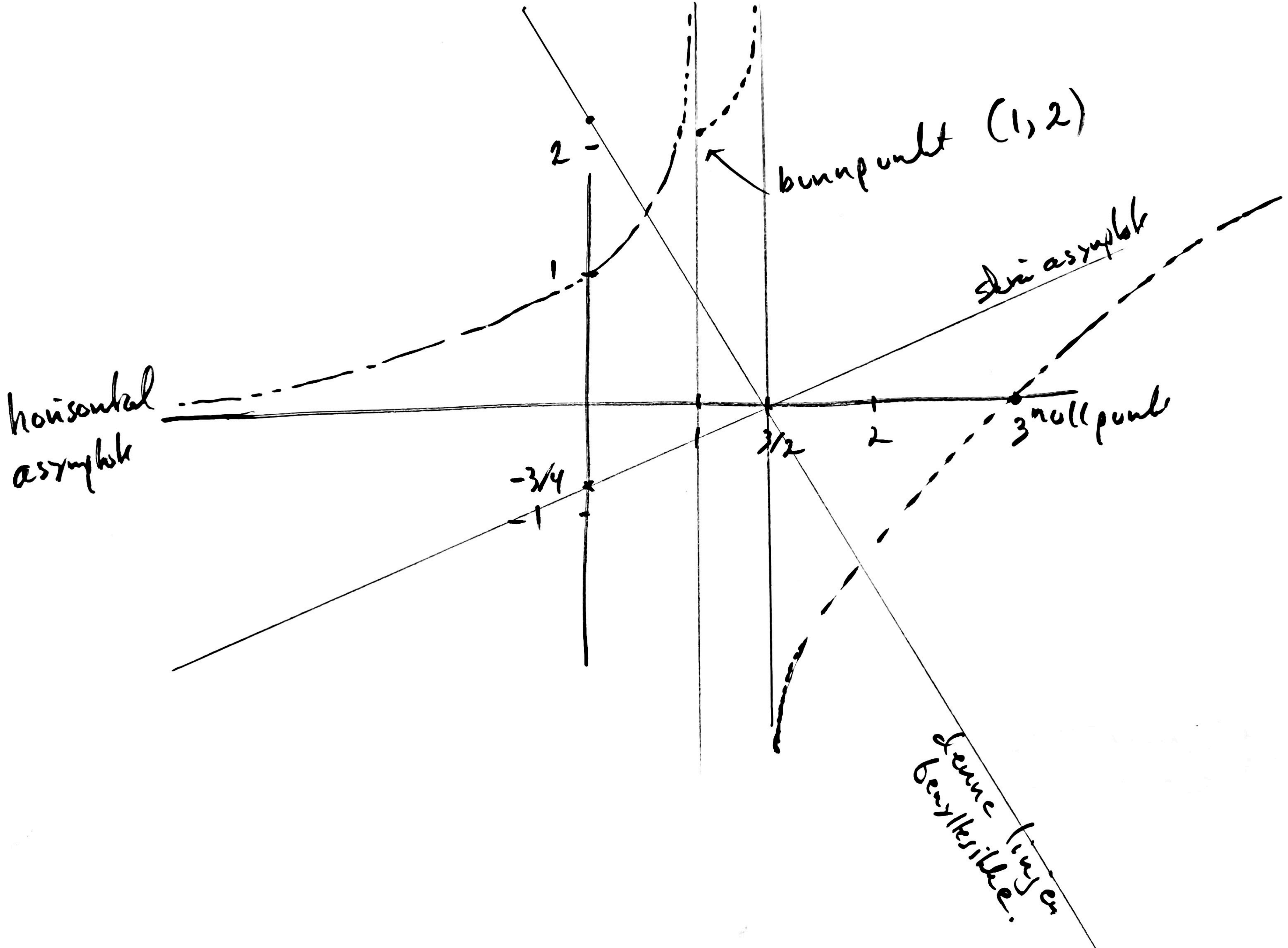
$g'(x) > 0$ for $x < 1$ (voksende)

$x > 1$: $\frac{1}{2}\left(1 + \frac{9}{(2x-3)^2}\right) = \frac{1}{2}\frac{1}{(2x-3)^2}\left[(2x-3)^2 + 9\right]$

> 0 for alle $x \geq 1$, $\neq 3/2$.

voksende.

$g(x)$ er voksende i $\langle -\infty, 1\rangle$ og $\langle 1, 3/2\rangle$ og $\langle 3/2, \infty\rangle$



horisontal asymptote
bunnpunkt (1, 2)
skrå asymptote
3 nullpunkt
2
1
-3/4
-1
1
3/2
2
denne linja er bevyktsikke.