30.01.23

12. <u>Vektorer</u>

1–5

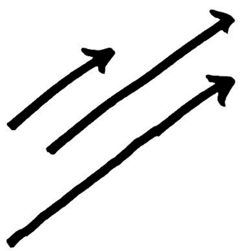
$\underline{V}$, $\vec{V}$, $\mathbb{V}$

like vektorer.

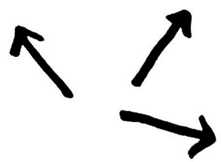
parallellforskyvde vektorer

To vektorer er like hvis
de har samme retning og
samme størrelse

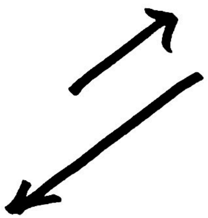
De kan da parallellforskyves
over i hverandre.



samme retning

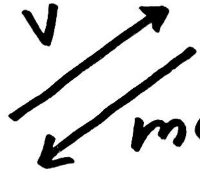


samme lengde



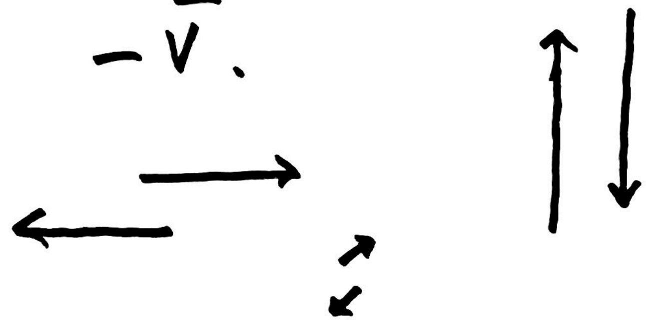
motsatt retning

$\vec{v}$



motsattvektoren til $\vec{v}$

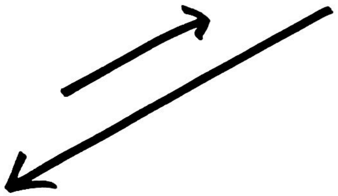
$-\vec{v}$.



$\vec{0}$ nullvektoren

- lengde 0

ingen retning (alle?)

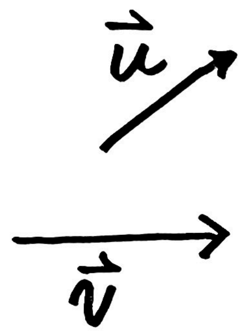


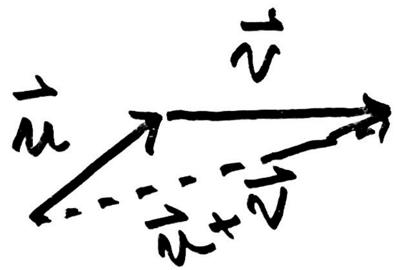
parallelle vektorer

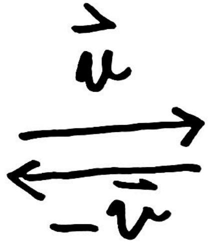
vektorene har samme eller motsatt retning.

Alle vektorer er parallelle til $\vec{0}$

Sum av vektor

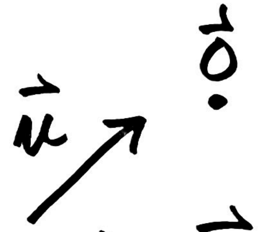






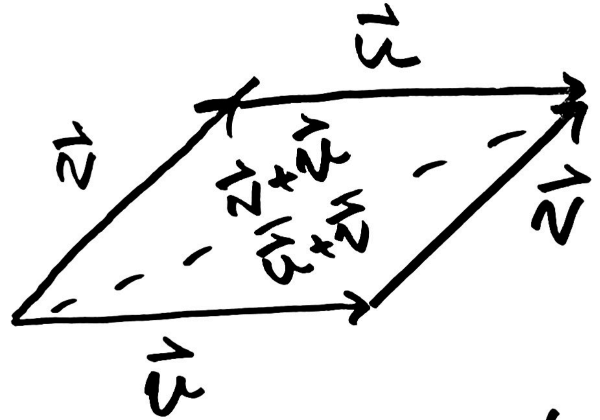
summen er $\vec{0}$

$$\vec{u} + (-\vec{u}) = \vec{0}$$



$$\vec{u} + \vec{0} = \vec{u}$$

$$\vec{0} + \vec{u} = \vec{u}$$



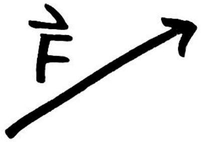
Sum av vektorer er kommutativ

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}.$$

og assosiativ

$$(\vec{u}+\vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$$

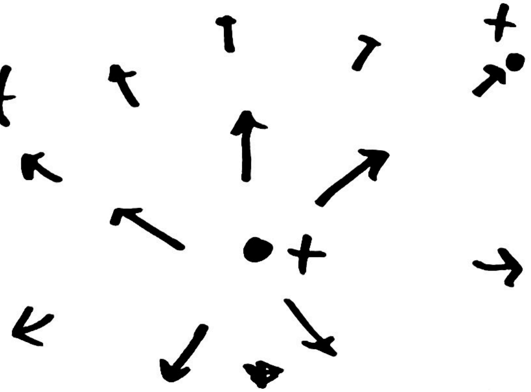
Vektorer forekommer naturlig i fysikk


$\vec{F}$ krefter

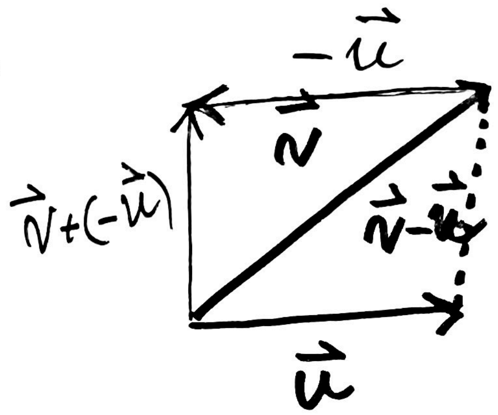
hastighet
(størrelsen: fart
og retning)

Kraftmoment
størrelse $r \cdot F$ når vinkelrette
i dette tilfellet er retningen ut av planet.

Elektrisk felt

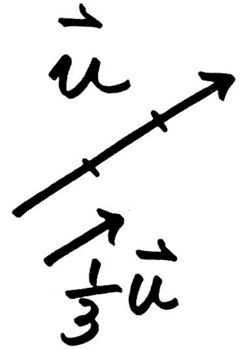


Differanse av vektorer

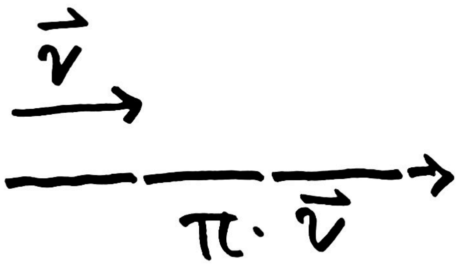


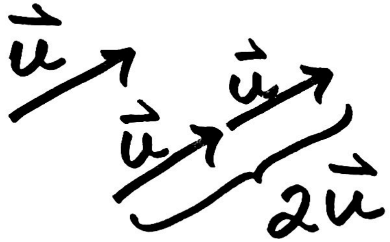
$$\vec{v}-\vec{u} = \vec{v} + \underbrace{(-\vec{u})}_{\text{motsatvektoren}}$$

$$(\vec{v}-\vec{u})+\vec{u}=\vec{v}$$

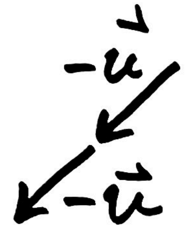


Skalere vektorer





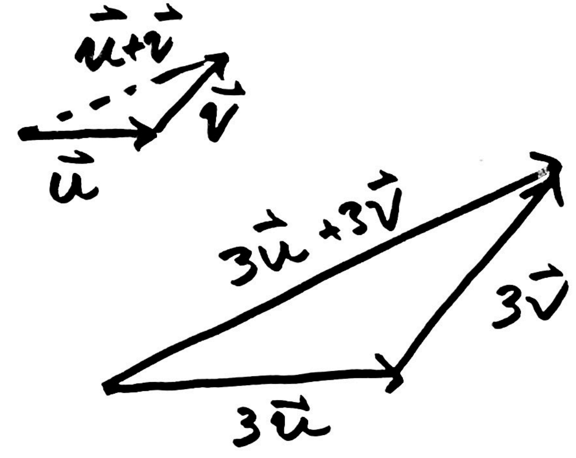
$$-2\vec{u} = 2(-\vec{u}) = -(2\vec{u})$$



$$r \cdot \vec{u} = \begin{cases} \text{samme retning, lengde } r \cdot \text{lengde til } \vec{u}, & r > 0 \\ \text{motsall retning, } - \text{ "} - & r < 0 \\ \vec{0} & r = 0 \end{cases}$$

skalarmult. av $\vec{u}$ med r

$$r(\vec{u} + \vec{v}) = r \cdot \vec{u} + r\vec{v}$$



$$1 \cdot \vec{u} = \vec{u}$$

$$s(r\vec{u}) = (s \cdot r)\vec{u}$$

$$(s+r)\vec{u} = s\vec{u} + r\vec{u}$$

skalarene er reelle tall

de skalerer vektorer.

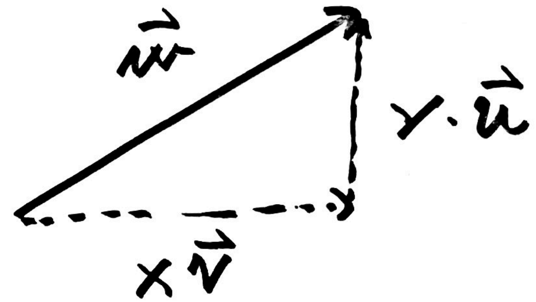
$$2\vec{a} + 3\vec{a} = (2+3)\vec{a} = 5\vec{a}$$

$$2(\vec{a} - 3\vec{b}) + 3(\vec{a} + \vec{b}) = 2\vec{a} + \overbrace{2(-3)\vec{b}}^{-6\vec{b}} + 3\vec{a} + 3\vec{b}$$

$$= \underline{5\vec{a} - 3\vec{b}}$$

To ikke-parallelle vektorer

$\vec{u}$ ↑ $\vec{v}$ →



For alle vektorer
$\vec{w}$ i planet
Så finnes det skalarer x og y slik at

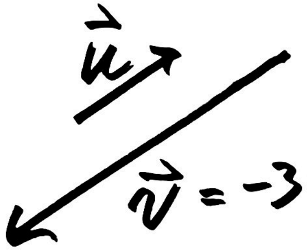
$$w = x \cdot \vec{v} + y\vec{u}$$

lineær kombinasjon.

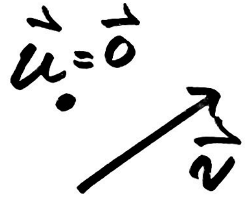
x, y <u>entydige</u>

$\vec{u}, \vec{v}$ <u>basis</u> for vektorer i planet.

$\vec{u}, \vec{v}$ parallelle hvis det finnes en skalar s
slik at $s\vec{u} = \vec{v}$ eller $s\vec{v} = \vec{u}$

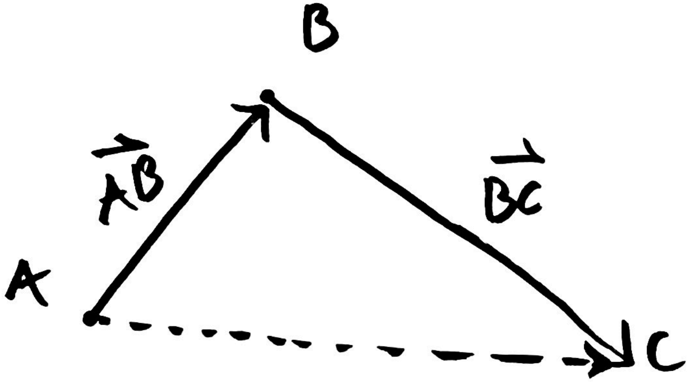


$\vec{v} = -3\vec{u}$
og $\vec{u} = -\frac{1}{3}\vec{v}$



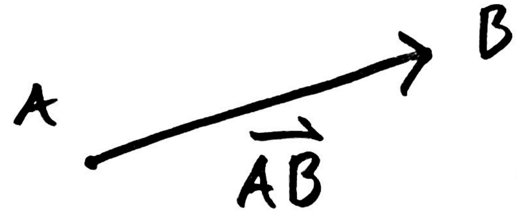
$\vec{u} = 0 \cdot \vec{v}$
$\vec{v}$ er ikke lik en skalering av $\vec{u}$.

Vektorer og punkt



$$\vec{AC} = \vec{AB} + \vec{BC}$$

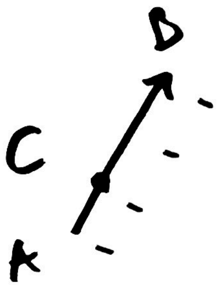
Vektoren fra A til B



$$\vec{AB} = -\vec{BA}.$$

eks

$$\vec{AB} - \vec{CB} = \vec{AB} + \overbrace{\vec{BC}}^{-\vec{CB}} = \vec{AC}.$$

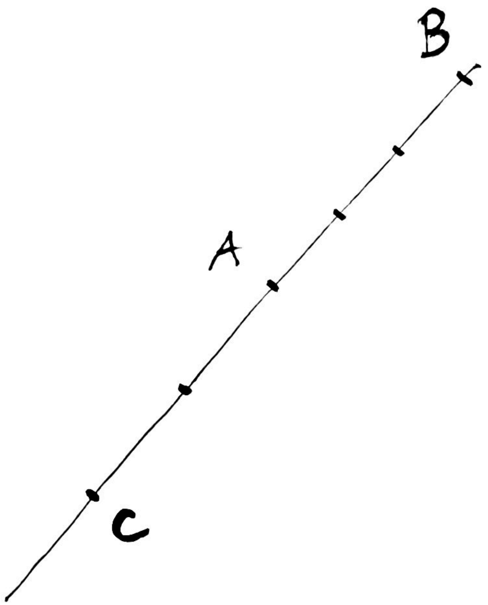


avstanden AC $= \frac{1}{2}$ avstanden CB

C mellom A og B.

Finn $\vec{AC}$.

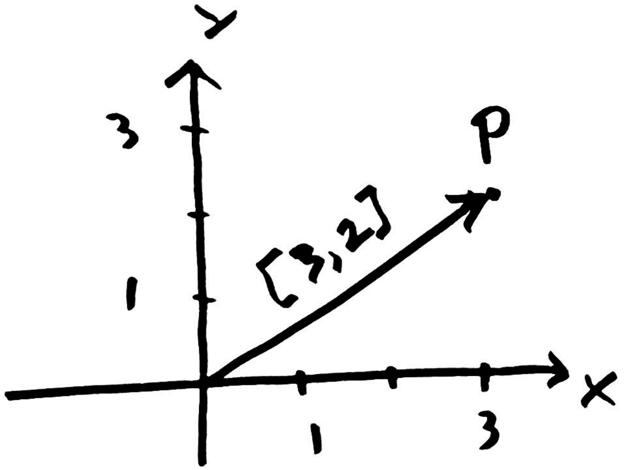
$$\vec{AC} = \underline{\frac{1}{3}}\vec{AB}$$



hvis c ikke ligger
mellom A og B får vi en
løsning til:

$$\vec{AC} = -\vec{AB}.$$

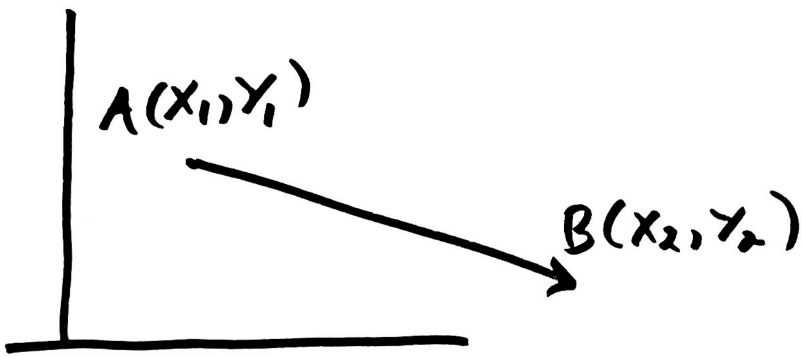
Vektorer på koordinatform



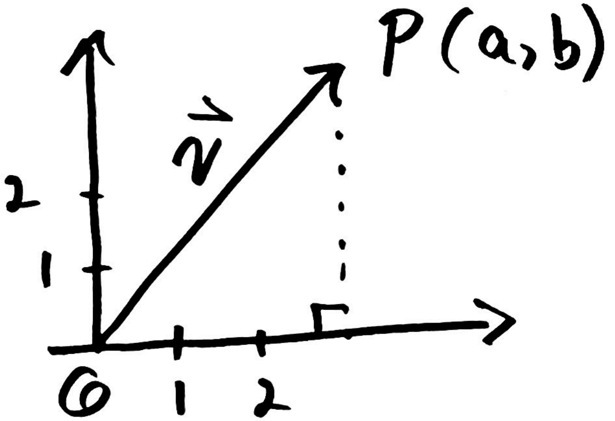
$\vec{u}$ parallellforskyver den
så den starter i origo $O:(0,0)$.
Endepunktet $P\ (x,y)$
koordinatene til $\vec{u}$:

$$\vec{u} = [x, y]$$

(firkantparenteser)



$\vec{AB} = [x_2 - x_1, y_2 - y_1]$



Lengden til $\vec{v}$ er lengden i koordinatsystemet

$\vec{v} = [a, b]$

Pytagoras

$|\vec{v}| = |[a,b]| = \sqrt{a^2 + b^2}$

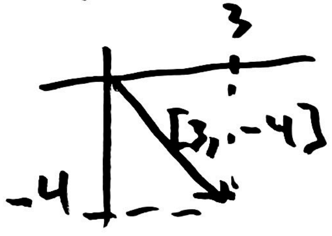
Euklidske normen.

Egenskaper til normer: $|\vec{v}| \geq 0$ og $|\vec{v}| = 0 \Leftrightarrow \vec{v} = 0$

$|s\vec{v}| = |s| \cdot |\vec{v}|$

$|\vec{u} + \vec{v}| \leq |\vec{u}| + |\vec{v}|$ trekantulikheten

$$|[3,-4]| = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = 5$$



$$[3s, 2s] = s\,[3,2]$$

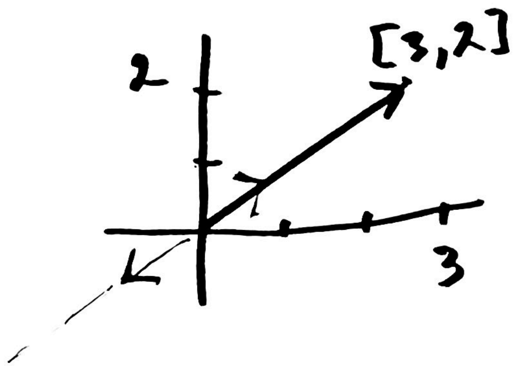
For hvilke s er lengden til $\vec{v}$ lik 1.

$$|[3s,2s]| = \sqrt{(3s)^2+(2s)^2} = \sqrt{s^2(9+4)} = 1$$

$$\sqrt{s^2}\sqrt{13} = 1$$

$$|s| = 1/\sqrt{13}$$

Så $\underline{s = \frac{1}{\sqrt{13}}}$ og $\underline{s = \frac{-1}{\sqrt{13}}}$



opg 1 $\vec{a} = 2\vec{x} - \vec{y}$ uttrykk $3\vec{a} - \vec{b}$ ved hjelp av

$\vec{b} = \vec{x} + 3\vec{y}$ $\vec{x}$ og $\vec{y}$.

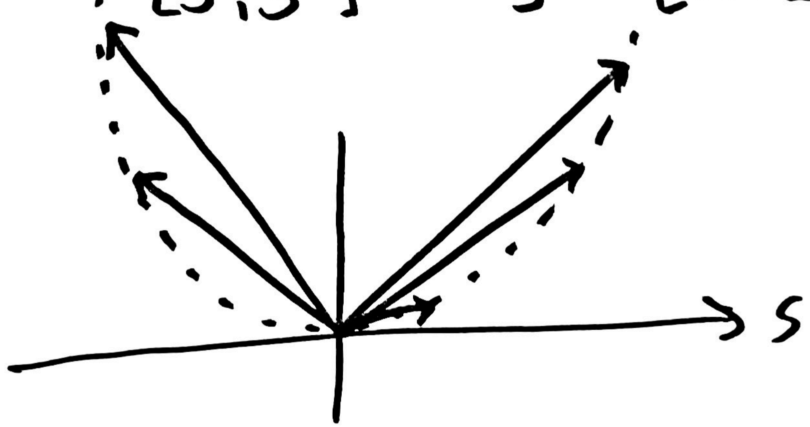
$$3\vec{a} - \vec{b} = 3(2\vec{x} - \vec{y}) - (\vec{x} + 3\vec{y}) = 6\vec{x} - 3\vec{y} - \vec{x} - 3\vec{y}$$

$$= 3 \cdot 2\vec{x} + 3(-1)\vec{y} + (-1)\vec{x} + (-1)3\vec{y}$$

$$= (6-1)\vec{x} + (-3-3)\vec{y} = \underline{5\vec{x} - 6\vec{y}}$$

OPG 2 Når er $[5, s^2]$ og $[1, 2]$ parallelle vektorer ?

$(r[x, y] = [rx, ry])$



$$t \overset{\neq 0}{[1, 2]} = [s, s^2]$$

$$[t, 2t] = [s, s^2]$$

$t = s$ x-koordinat

$2t = s^2$ y— " —

Setter inn $t = s$: $2s = s^2$

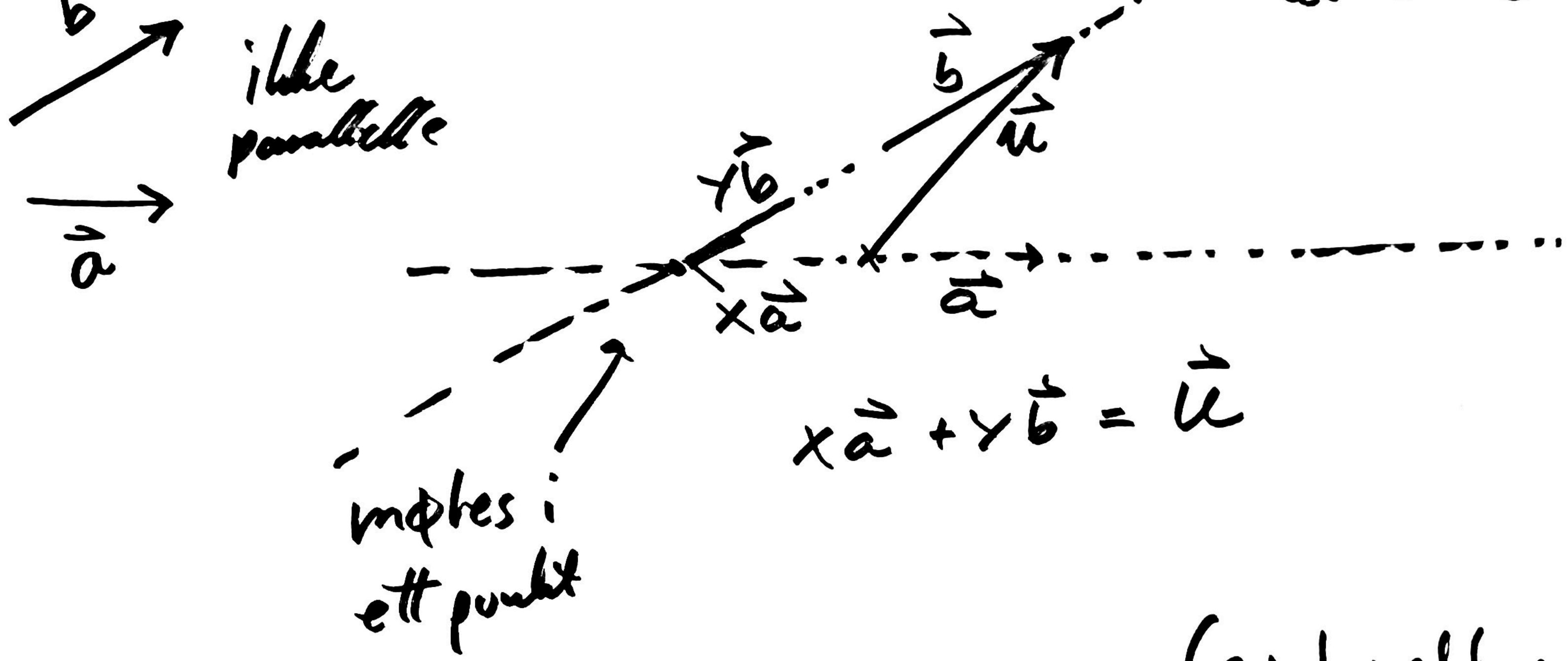
$s^2 - 2s = 0$

$s(s-2) = 0$

To løsninger $s = 0$ $[0,0]$

$\underline{s = 2}$ $[2,4] = 2[1,2]$.

$\vec{u} = x\vec{a} + y\vec{b}$ lineær kombinasjon av $\vec{a}$ og $\vec{b}$.



$x\vec{a} + y\vec{b} = \vec{u}$

(søylevektorer)

$$x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}$$

$$x[1,2] + y[1,-1] = [5,1]$$

$$[x+y,\ 2x-y] = [5,1]$$

$L1 \quad x + y = 5$

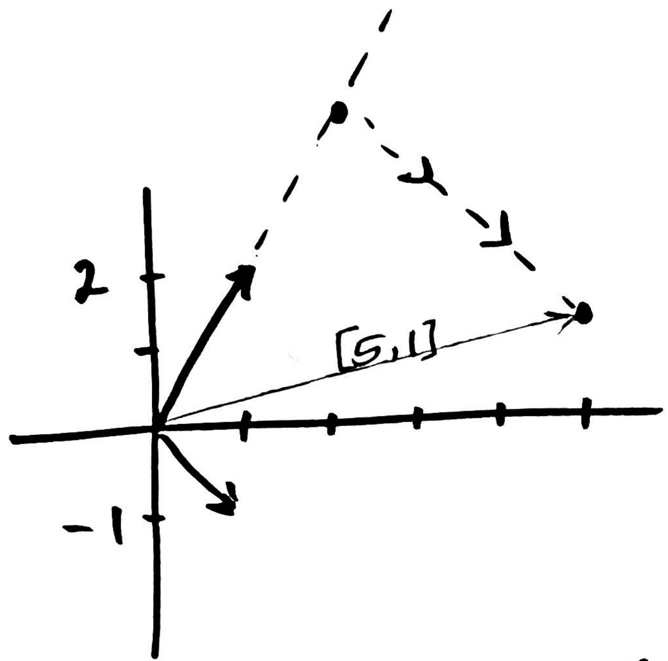
$L2 \quad 2x - y = 1$

L1+L2

$3x = 6$ så $x = 2$

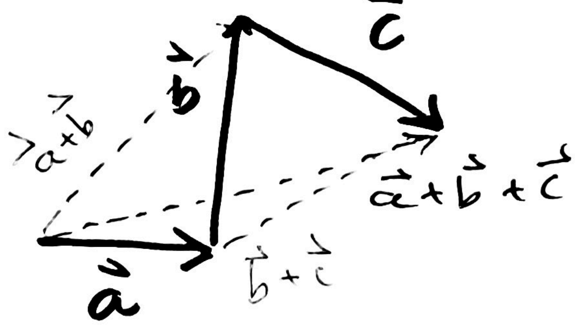
Setter inn for x i L1

$2 + y = 5$ så $y = 3$.



$(\vec{a} + \vec{b}) + \vec{c} = a + (\vec{b} + \vec{c})$

Assosiativitet til sum



opg Forenkl

$$\vec{AB} + \vec{CB} - \vec{AC}$$

$$\vec{AB} + \vec{CB} + \vec{CA}$$

$$\underbrace{\vec{CA} + \vec{AB}}_{\vec{CB}} + \vec{CB} = \underline{2\vec{CB}}$$

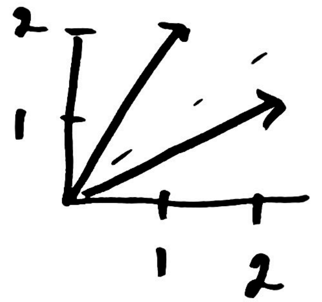
$$\left(\begin{aligned} \vec{DE} + \vec{EF} &= \vec{DF} \\ -\vec{DE} &= \vec{ED} \end{aligned} \right)$$

Finn x og y slik at

$$x + 2y = 20$$
$$2x + y = 19$$

$$x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}20\\19\end{bmatrix}$$

Forventer x, y ca like store
og $y > x$.



$x = 20 - 2y$ setter inn i L2

$$2(20-2y) + y = 19$$
$$40 - 4y + y = 19, \quad 21 = 3y$$

Så $\underline{\underline{y = 7}}$ $\quad x = 20 - 2y = \underline{\underline{6}}$