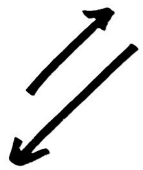
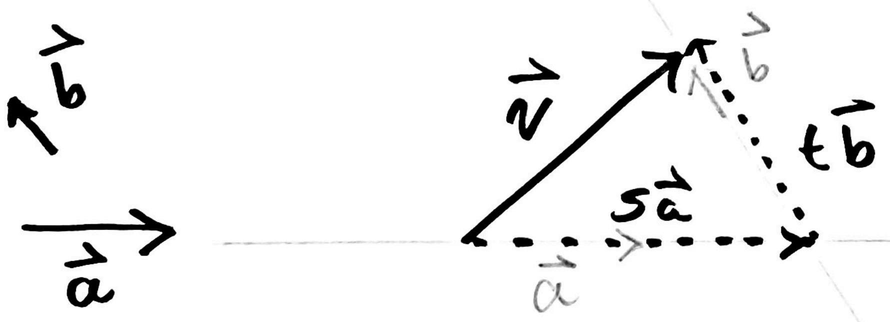
3 feb. 2023



parallelle : samme eller motsatt retning
eller minst én av den er $\vec{0}$

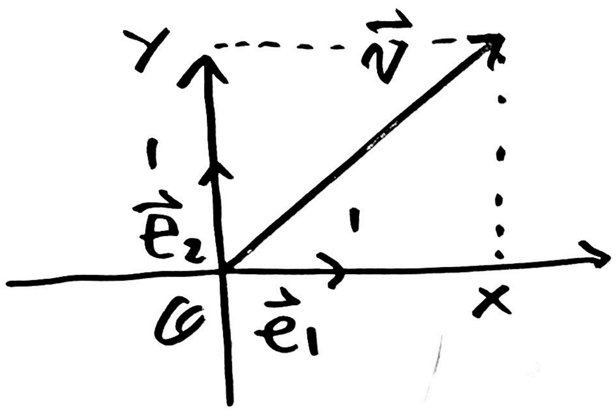
$\vec{u}, \vec{v}$, anta $\vec{u} \neq \vec{0}$
$\vec{u}, \vec{v}$ parallelle $\Leftrightarrow$ finnes en skalar t
$t \cdot \vec{u} = \vec{v}$

To ikke-parallelle vektorer utspenner alle vektorer i planet



$\vec{v} = s\vec{a} + t\vec{b}$
s, t er entydige

$\vec{v} = [s, t]^{\text{basis } \vec{a}, \vec{b}}$



standard basis

$[x,y] = x\vec{e}_1 + y\vec{e}_2$

Euklidsk norm

$|[x,y]| = \sqrt{x^2+y^2}$ (ved Pythagoras)

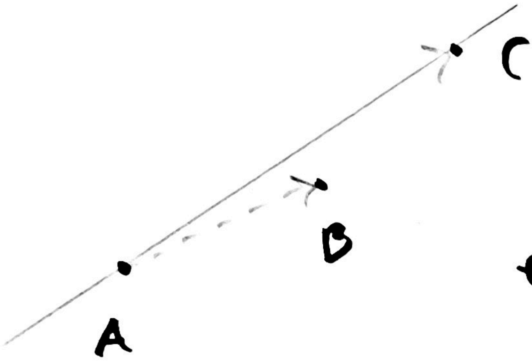
lengden

Til orientering : $\mathbb{R}^2$, elementer $[x,y]$ planet

$\mathbb{R}^3$, $[x,y,z] = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$ rommet

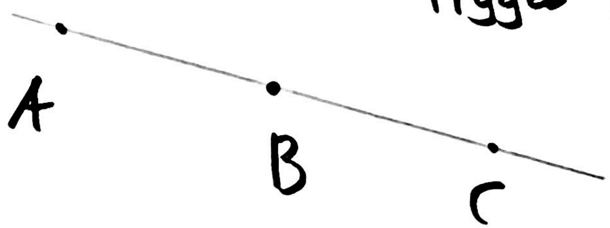
$\vdots$

$\mathbb{R}^n$, $[x_1,\dots,x_n] = x_1\vec{e}_1 + x_2\vec{e}_2 + \cdots + x_n\vec{e}_n$ n-dimensjonalt vektorrom.



liggar ikkje på en linje

$\vec{AB}$ og $\vec{AC}$ er ikkje parallelle.



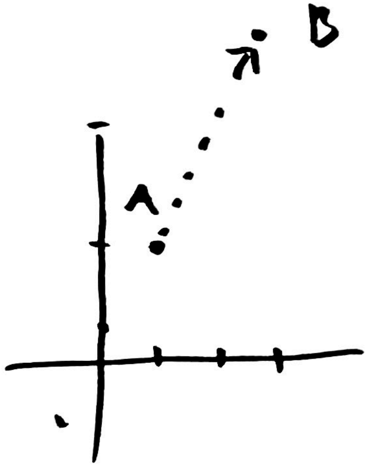
liggar på en linje.

$\vec{AB} \parallel \vec{AC}$

parallelle

Eks $A(1,2)$ $B(3,5)$ og $C(-1,-1)$

Liggar de tre punktene på en linje?



$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= [3,5] - [1,2]$

$= \underline{[2,3]}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$= [-1,-1] - [1,2]$

$= [-2,-3]$

$= -\vec{AB}$ parallelle

Så A, B og C liggar på en linje.

opg

Ligger $A(2,3)$ $B(4,0)$ og $C(7,-2)$ på en linje?

Sjekker om $\vec{BA}$ og $\vec{BC}$ er parallelle

$\vec{BA} = \vec{OA} - \vec{OB} = [2,3]-[4,0] = [-2,3]$

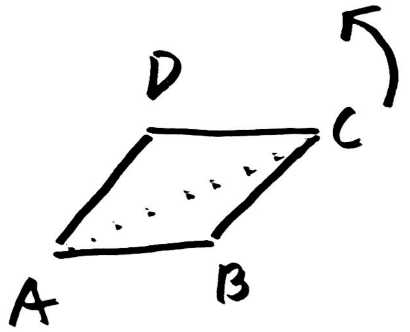
$\vec{BC} = \vec{OC} - \vec{OB} = [7,-2]-[4,0] = [3,-2]$

$t[-2,3] = [3,-2] \Leftrightarrow -2t = 3 \quad : t = -3/2$

og $3t = -2 \quad : t = -2/3$

ingen løsning. ingen felles løsning

$\vec{BA}$ og $\vec{BC}$ er ikkje parallelle, så A, B og C ligger ikkje på en linje.



parallellogram
Motstående sider er parallelle og
like lange

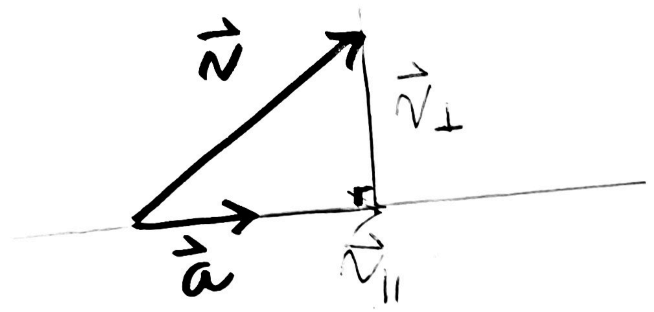
Gitt koordinater til A, B og D i et parallellogram.
Finn C.

$\vec{AB} = \vec{DC}$
$\vec{BC} = \vec{AD}$

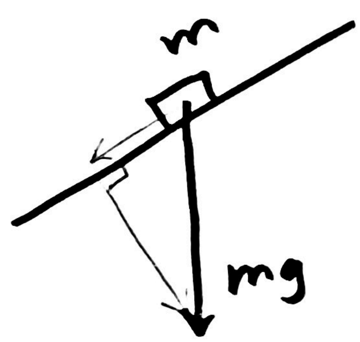
$$\vec{AC} = \vec{AB} + \underbrace{\vec{BC}}_{\vec{AD}}$$

$$\vec{AC} = \vec{AB} + \vec{AD}$$

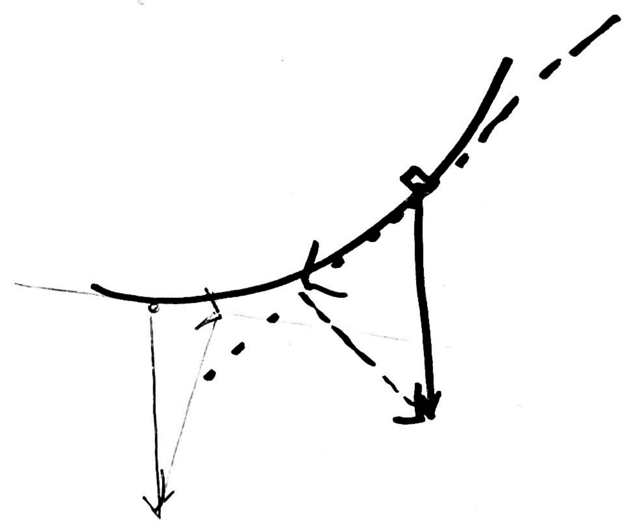
$$\vec{OC} = \vec{OA} + \vec{AC} = \underbrace{\vec{OA} + \vec{AB}}_{\vec{OB}} + \vec{AD}.$$



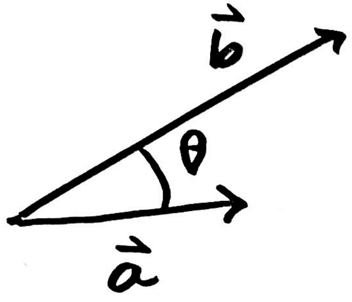
dekomponere $\vec{v}$ som en sum
av en vektor parallell til $\vec{a}$: $\vec{v}_\parallel$
og en vektor vinkelrett på $\vec{a}$: $\vec{v}_\perp$



skrått plan



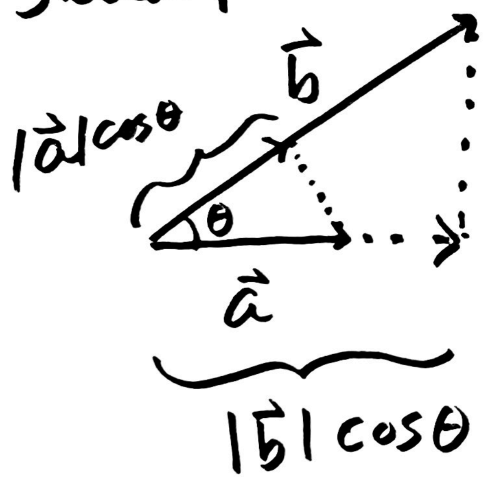
13. Skalarproduktet (prikk-produkt)

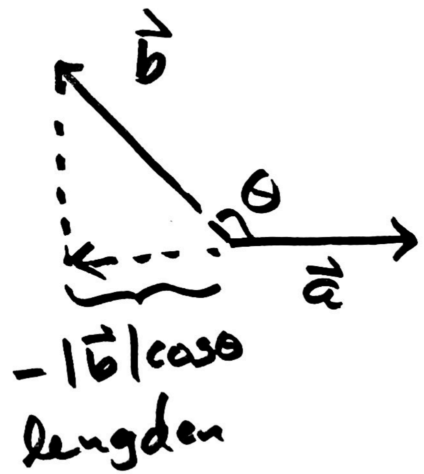


θ : teta (gresk)

$0 \leq \theta \leq 180^\circ$

skalarproduktet : $\vec{a} \cdot \vec{b} = \underbrace{|\vec{a}||\vec{b}|\cos(\theta)}_{\text{skalar}}$





$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ symmetrisk i $\vec{a}$ og $\vec{b}$

$|\vec{a}| = 3$ $\quad |\vec{b}| = 5$ $\quad$ vinkelen mellom dem er $135°$.

Da er $\quad \vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(135°)$

$$= 3 \cdot 5 \cdot \frac{-1}{\sqrt{2}} = \underline{\frac{-15}{\sqrt{2}}}$$

$\cos(135°) =$
$-\cos(45°)$
$= -1/\sqrt{2}$

Hva er vinkelen mellom $\vec{a}$ og $\vec{b}$

når $\quad |\vec{a}| = 5 \quad |\vec{b}| = 10 \quad \vec{a} \cdot \vec{b} = 25$

$$25 = \vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\theta) = 5 \cdot 10 \cdot \cos\theta$$

så $\quad \cos\theta = \frac{25}{5 \cdot 10} = \frac{1}{2}$

$$\theta = \arccos\left(\tfrac{1}{2}\right)$$
$$= \underline{60°}$$

$\vec{a}$ og $\vec{b}$ er parallelle $\Leftrightarrow$ $|\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}|$

$(|\cos\theta| = 1,\ \theta = 0°$ eller $180°)$

$$-|\vec{a}|\cdot|\vec{b}| \le \vec{a}\cdot\vec{b} \le |\vec{a}|\cdot|\vec{b}|$$

$\vec{a}\cdot\vec{b} = 0 = |\vec{a}||\vec{b}|\cos\theta$

$\theta = 90°$ eller minst én av $\vec{a}$ og $\vec{b}$ er lik $\vec{0}$

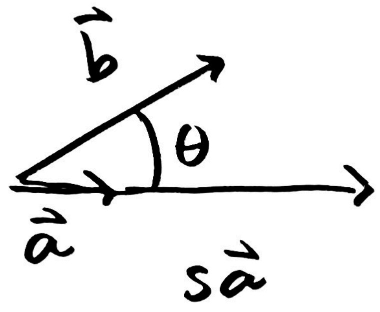
$\vec{a}$ og $\vec{b}$ er <u>ortogonale</u> hvis $\vec{a}\cdot\vec{b} = 0$

$\vec{a}\cdot\vec{b}$ er lineær i $\vec{a}$ og i $\vec{b}$

$$(s\vec{a})\cdot\vec{b} = s(\vec{a}\cdot\vec{b})$$

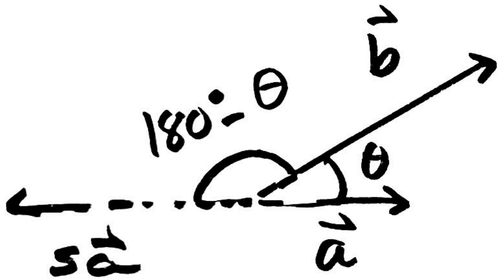
$$(\vec{a}_1+\vec{a}_2)\cdot\vec{b} = \vec{a}_1\cdot\vec{b} + \vec{a}_2\cdot\vec{b}$$

$s \geq 0$



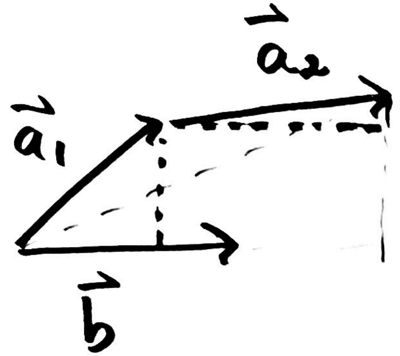
$$
\begin{aligned}
&(s\vec{a})\cdot\vec{b} \\
&= |s\vec{a}||\vec{b}|\cos\theta \\
&= s|\vec{a}|\cdot|\vec{b}|\cos\theta \\
&= s(\vec{a}\cdot\vec{b}).
\end{aligned}
$$

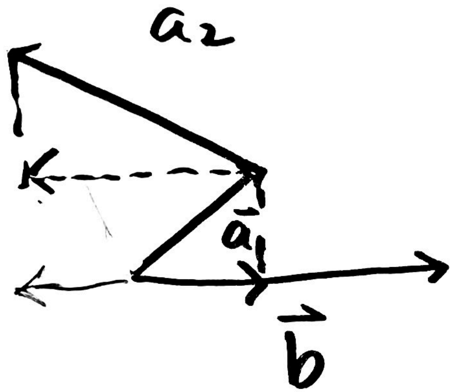
$s < 0$

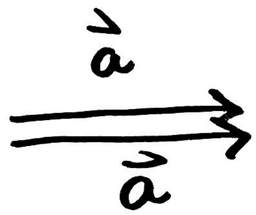


$$
\begin{aligned}
&(s\vec{a})\cdot\vec{b} \\
&= |s\vec{a}|\cdot|\vec{b}|\underbrace{\cos(180°-\theta)}_{-\cos(\theta)} \\
&= \underbrace{|s|}_{-s}|\vec{a}||\vec{b}| \\
&= |\vec{a}||\vec{b}|\cos\theta = s(\vec{a}\cdot\vec{b})
\end{aligned}
$$

$$(\vec{a}_1+\vec{a}_2)\cdot\vec{b} = \vec{a}_1\cdot\vec{b} + \vec{a}_2\cdot\vec{b}$$



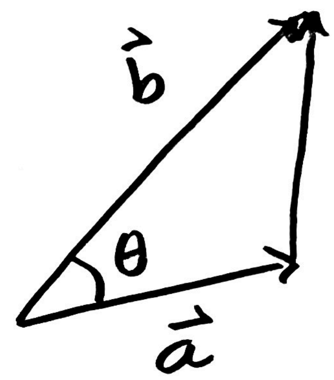




$\theta = 0°$

$\vec{a}\cdot\vec{a}$
$= |\vec{a}|^2$

Cosinussetning



$\vec{b} - \vec{a} = \vec{c}$

$a = |\vec{a}|$
$b = |\vec{b}|$
$c = |\vec{c}|$

$$|\vec{c}|^2 = |\vec{b} - \vec{a}|^2 = (\vec{b} - \vec{a}) \cdot (\vec{b} - \vec{a})$$

$$= \vec{b} \cdot (\vec{b} - \vec{a}) - \vec{a} \cdot (\vec{b} - \vec{a})$$

$$= \vec{b} \cdot \vec{b} - \vec{b} \cdot \vec{a} - \underbrace{\vec{b} \cdot \vec{a}}_{\vec{a} \cdot \vec{b}} - (-\vec{a} \cdot \vec{a})$$

$$|\vec{b}|^2 - 2\vec{b} \cdot \vec{a} + |\vec{a}|^2$$

$$\underline{c^2 = a^2 + b^2 - 2ab\cos\theta}$$

Neste uke: $[x_1, y_1] \cdot [x_2, y_2] = x_1 \cdot x_2 + y_1 \cdot y_2.$

Øving

opg. $|\vec{a}| = 2$ $\quad |\vec{b}| = 3$ $\quad \vec{a}\cdot\vec{b} = -5$

Finn $|2\vec{a}+\vec{b}|$

$|2\vec{a}+b|^2 = (2\vec{a}+\vec{b})\cdot(2\vec{a}+\vec{b})$

$2\vec{a}\cdot 2\vec{a} + \vec{b}\cdot\vec{b} + 2\vec{a}\cdot\vec{b} + \vec{b}\cdot 2\vec{a}$

$4|\vec{a}|^2 + |\vec{b}|^2 + 4\,\vec{a}\cdot\vec{b}$

$4\cdot 2^2 + 3^2 + 4(-5)$

$16 + 9 - 20 = 25 - 20 = 5$

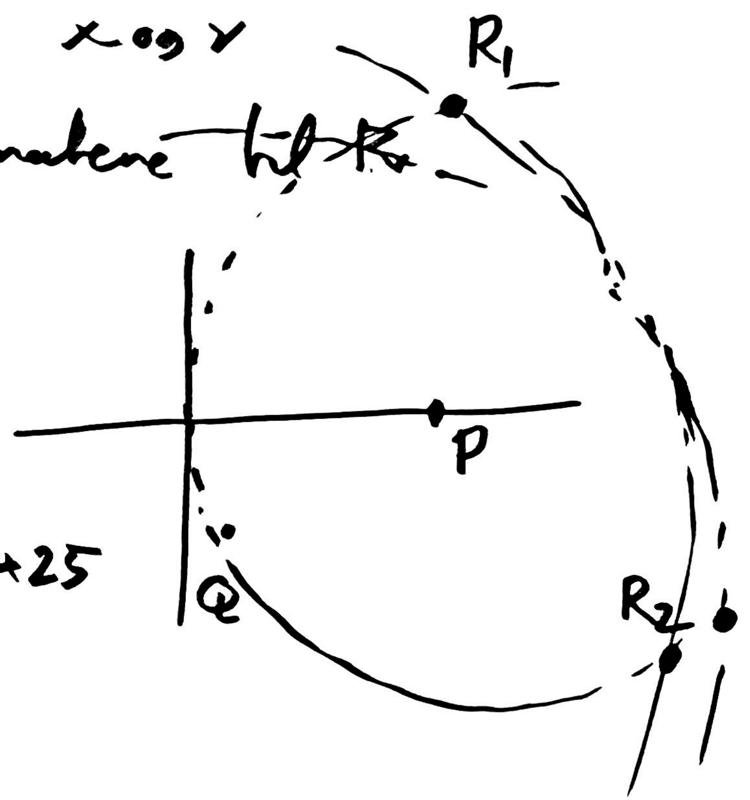
$|2\vec{a}+\vec{b}| = \underline{\sqrt{5}}$

opg 12.159 $P(5,0)$ og $Q(1,-3)$ $R(x,y)$

$|\vec{PR}| = 5$ $|\vec{QR}| = 5\sqrt{2}$

a) $\vec{PR}$ og $\vec{QR}$ uttrykt ved x og y

b) Finne x og y koordinatene ~~til R~~



a) $\vec{PR} = \vec{OR} - \vec{OP} = [x-5, y]$

$\vec{QR} = \vec{OR} - \vec{OQ} = [x-1, y+3]$

$|\vec{PR}|^2 = 5^2 = (x-5)^2 + y^2 = x^2 + y^2 - 10x + 25$

$|\vec{QR}|^2 = (\sqrt{25})^2 = 50 = (x-1)^2 + (y+3)^2$

$= x^2 + y^2 - 2x + 6y + 1 + 9$

Giv $x^2 + y^2 - 10x = 0$

og $x^2 + y^2 - 2x - 6y = 50 - 10 = 40$

+

$$\underbrace{10x}_{x^2+y^2} - 2x + 6y = 40$$

$$\Leftrightarrow \quad 8x - 6y = 40 \qquad \text{deler med 2}$$

$$\underline{4x - 3y = 20}$$

$$3y = 4x - 20 \quad \text{så} \quad y = -\left(\frac{4x-20}{3}\right)$$

setter inn i $\quad x^2 + y^2 - 10x = 0$

$$x^2 + \left(\frac{4x-20}{3}\right)^2 - 10x = 0 \quad \Big| \cdot 3^2$$

$$9x^2 + 16x^2 - 160x + 400 - 90x = 0$$

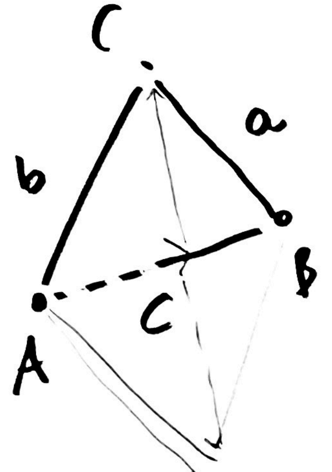
$$25x^2 - 250x + 400 = 0 \quad \Big| /25$$

$$x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0 \quad \text{gir} \quad \underline{x=2} \quad \underline{x=8}$$

$(2, +4)$ og $(8, -4)$

Generelt tilfelle



Anta A, B
a b er kjente.

Hva er C? typisk to muligheter

skalarproduktet
$\vec{u} \cdot \vec{AB} = 0$ ortogonale
$\vec{u} \neq \vec{0}$. $|\vec{u}| = 1$

Velger en vektor $\vec{u}$ vinkelrett på $\vec{AB}$

Forsøker å uttrykke $\vec{AC}$ som en lineær kombinasjon av $\vec{AB}$ og $\vec{u}$.

$$\vec{AC} = x\vec{u} + y\vec{AB}.$$

$$\vec{BC} = \vec{AC} - \vec{AB} = x\vec{u} + (y-1)\vec{AB}.$$

$$|\vec{AC}|^2 = b^2 = (x\vec{u} + y(\vec{AB})) \cdot (x\vec{u} + y(\vec{AB}))$$

$$= x^2 + y^2 \cdot c^2 + x \cdot y \underbrace{u \cdot \vec{AB}}_{0}$$

$$|\vec{BC}|^2 = a^2 = |x\vec{u} + (y-1)\vec{AB}|^2$$
$$= x^2 + (y-1)^2 c^2 + 0$$

L1 $\quad x^2 + y^2c^2 = b^2$

L2 $\quad x^2 + y^2c^2 + (-2y+1)c^2 = a^2$

$(x^2 + y^2c^2 - 2yc^2 = a^2 - c^2.)$

L1−L2 : $\quad (-2y+1)c^2 = a^2 - b^2$

$$-2yc^2 = a^2 - b^2 - c^2$$

$$y = \frac{a^2-b^2-c^2}{-2c^2} = \frac{b^2+c^2-a^2}{2c^2}$$

L1 giv: $\quad x = \pm\sqrt{b^2 - \frac{(b^2+c^2-a^2)^2}{4c^2}}$