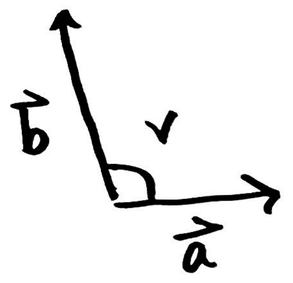
06.02.23 13.1-4 Skalarprodukt

$\vec{a} \bullet \vec{b}$ skalar (prikkproduktet)



$$\vec{a} \bullet \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos v$$

$$\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a} \quad \text{symmetrisk}$$

$0 \leq v \leq \pi$

$$-|\vec{a}||\vec{b}| \leq \vec{a} \bullet \vec{b} \leq |\vec{a}||\vec{b}|$$

Skalarproduktet er lineært i begge vektorene

$$\vec{a} \cdot \vec{b} = -3$$

$|\vec{a}| = 2$

$|\vec{b}| = 3$

Hva er vinkelen mellom $\vec{a}$ og $\vec{b}$?

$$\cos v = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{-3}{2 \cdot 3} = \frac{-1}{2}$$

$$v = \arccos\left(\frac{-1}{2}\right) = \underline{\frac{2\pi}{3}} \quad (120^\circ)$$

oppgave $|\vec{a}| = 2$

$|\vec{b}| = 3$

Hva kan vi si om vektoren (vinkelen mella de)

hvis

$\vec{a} \cdot \vec{b} = 6 \; (= |\vec{a}| \cdot |\vec{b}|)$ $\quad v = 0$

$\vec{a} \cdot \vec{b} = -6$ $\quad v = \pi$

$\vec{a} \cdot \vec{b} = 0$ $\quad v = \pi/2$

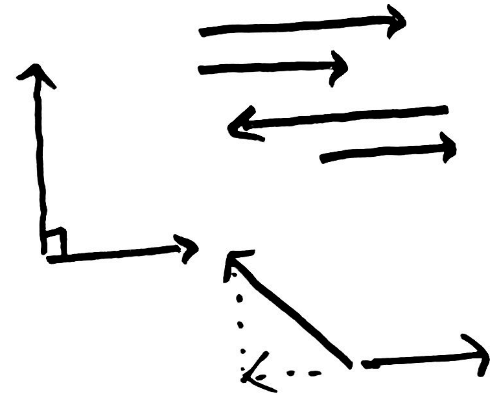
$\vec{a} \cdot \vec{b} < 0$ $\quad \frac{\pi}{2} < v \leq \pi$

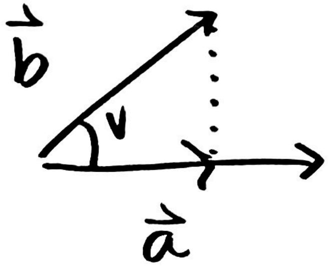
$\vec{a} \cdot \vec{b} = 7$ ikke mulig

$\vec{a} \cdot \vec{b} = 1$ $\quad \cos v = \frac{1}{6}$

$v = \arccos\left(\frac{1}{6}\right) = 80.406^\circ$

≈ 1.403 rad.

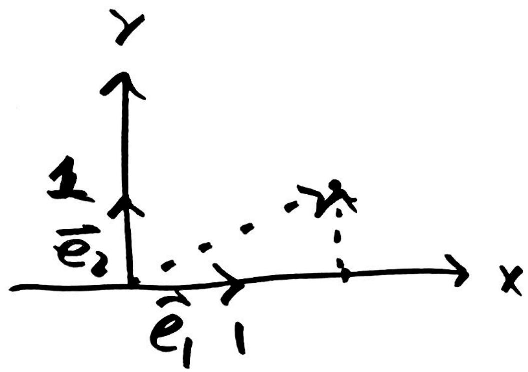




$\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$ = { lengden til komponenten til $\vec{b}$ langs $\vec{a}$, med fortegn
+ hvis samme retning
− hvis motsatt retning.

Skalarprodukt på koordinatform

$$[x,y] = x\vec{e}_1 + y\vec{e}_2$$
$$= x[1,0] + y[0,1]$$

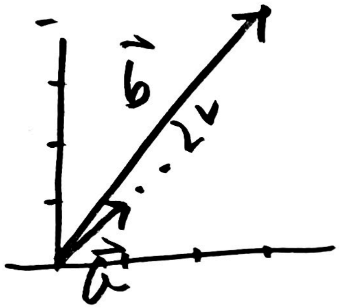


$$\vec{e}_1 \cdot \vec{e}_1 = |\vec{e}_1|^2 = 1$$
$$\vec{e}_2 \cdot \vec{e}_2 = |\vec{e}_2|^2 = 1$$

$$\vec{e}_1 \cdot \vec{e}_2 = \vec{e}_2 \cdot \vec{e}_1 = 0$$

$$[x_1, y_1] \cdot [x_2, y_2] = (x_1\vec{e}_1 + y_1\vec{e}_2) \cdot (x_2\vec{e}_1 + y_2\vec{e}_2)$$
$$= x_1 x_2 \underbrace{\vec{e}_1 \cdot \vec{e}_1}_{1} + x_1 y_2 \underbrace{\vec{e}_1 \cdot \vec{e}_2}_{0} + y_1 x_2 \underbrace{\vec{e}_2 \cdot \vec{e}_1}_{0} + y_1 y_2 \underbrace{\vec{e}_2 \cdot \vec{e}_2}_{1}$$
$$= x_1 \cdot x_2 + y_1 \cdot y_2$$

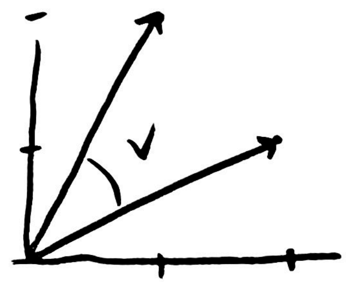
$$[3,4]\cdot[1,1] = 3\cdot 1 + 4\cdot 1 = 7$$



$$\cos v = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{7}{\sqrt{9+4^2}\sqrt{1+1}} = \frac{7}{5\sqrt{2}}$$

$$v = \arccos\left(\frac{7}{5\sqrt{2}}\right) = 8.13^\circ$$

Opg. Finn skalarproduktet mellom $\vec{a} = [1,2]$ og $\vec{b} = [2,1]$ og vinkelen mellom dem.



$$\vec{a}\cdot\vec{b} = [1,2]\cdot[2,1] = 1\cdot 2 + 2\cdot 1 = \underline{4}$$

$$|\vec{a}| = |\vec{b}| = \sqrt{1+4} = \sqrt{5}$$

$$\cos v = \frac{4}{\sqrt{5}\cdot\sqrt{5}} = \frac{4}{5} = 0.8$$

$$v = \arccos(0.8) = \underline{36.87^\circ}$$

Når er $\underset{\vec{a}}{[2,3]}$ og $\underset{\vec{b}}{[s, s+1]}$ ortogonale?

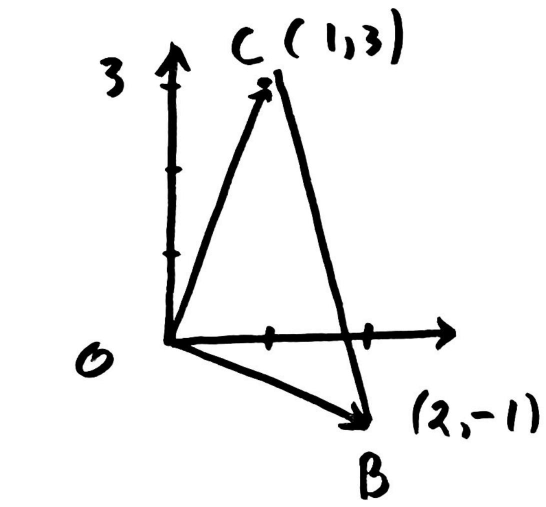
Det skjer når $\vec{a} \cdot \vec{b} = 0$

$$[2,3] \cdot [s, s+1] = 2s + 3(s+1) = 0$$
$$= 5s + 3 = 0$$

Så vektorene $\vec{a}$ og $\vec{b}$ er ortogonale når $s = \frac{-3}{5} = \underline{-0.6}$

$[a, b]$ og $[b, -a]$ er ortogonale vektorer.

$[-b, a]$



Finn vinklene til trekanten

$\vec{OB} = [2,-1]$

$\vec{OC} = [1,3]$

$$\cos(\angle O) = \frac{\vec{OB} \cdot \vec{OC}}{|\vec{OB}| \cdot |\vec{OC}|}$$

$$= \frac{2-3}{\sqrt{5}\sqrt{10}} = \frac{-1}{\sqrt{5}\sqrt{2 \cdot 5}} = \underline{\frac{-1}{5\sqrt{2}}}$$

$$\angle O = \underline{98.13^\circ}$$

$$\cos(\angle B) = \frac{\vec{BO} \cdot \vec{BC}}{|\vec{BO}| \cdot |\vec{BC}|}$$

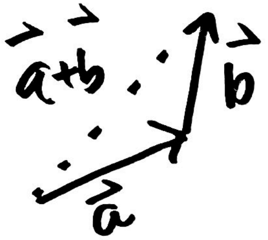
$$= \frac{2+4}{\sqrt{5}\sqrt{17}}$$

$\vec{BO} = -\vec{OB} = [-2,1]$

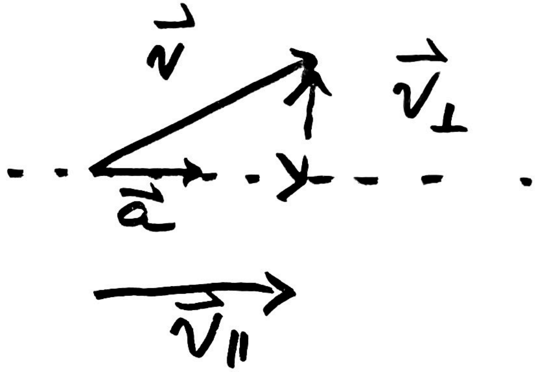
$\vec{BC} = \vec{OC} - \vec{OB} = [-1,4]$

$$\angle B = \arccos \frac{6}{\sqrt{85}} = \underline{49.40^\circ}$$

$$\angle C = 180^\circ - \angle O - \angle B = \underline{32.47^\circ}$$



Sum

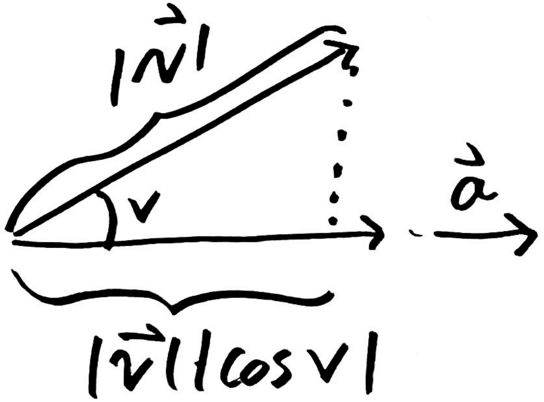


$$\vec{v} = \vec{v}_\parallel + \vec{v}_\perp$$

$$\vec{v}_\parallel = \frac{\vec{v}\cdot\vec{a}}{|\vec{a}|^2}\,\vec{a}$$

komponenten til $\vec{v}$ langs $\vec{a}$

$$\vec{v}_\perp = \vec{v} - \vec{v}_\parallel$$



$$\vec{v}_\parallel = \frac{\vec{v}\cdot\vec{a}}{|\vec{a}|}\cdot\frac{\vec{a}}{|\vec{a}|}$$

retningen til $\vec{a}$

$\vec{v} = [3,4]$ $\qquad$ $\vec{a} = [1,1]$.

Finn komponenten til $\vec{v}$ langs $\vec{a}$ og vinkelrett på $\vec{a}$

$$\vec{v}_{\parallel} = \frac{\vec{a}\cdot\vec{v}}{|\vec{a}|^2}\vec{a} = \frac{3+4}{1+1}[1,1] = \underline{\tfrac{7}{2}[1,1]}.$$

$$\vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} = [3,4] - (3.5)[1,1]$$
$$= [-0.5, 0.5] = \underline{\tfrac{1}{2}[-1,1]}$$

opg $\quad \vec{v} = [5,2]$ $\qquad$ $\vec{a} = [-3,2]$

Finn $\vec{v}_{\parallel}$ og $\vec{v}_{\perp}$

$$\vec{v}_{\parallel} = \frac{\vec{v}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} = \frac{-15+4}{9+4}[-3,2] = \frac{-11}{13}[-3,2]$$
$$= \left[\frac{33}{13}, -\frac{22}{13}\right]. \sim [2{,}538, -1{,}692]$$

$$\vec{v} - \vec{v}_{\parallel} = [5,2] - \left[\frac{33}{13}, \frac{-22}{13}\right]$$

$$= \frac{1}{13}[\underbrace{65-33}_{32}, \underbrace{26-(-22)}_{48}]$$

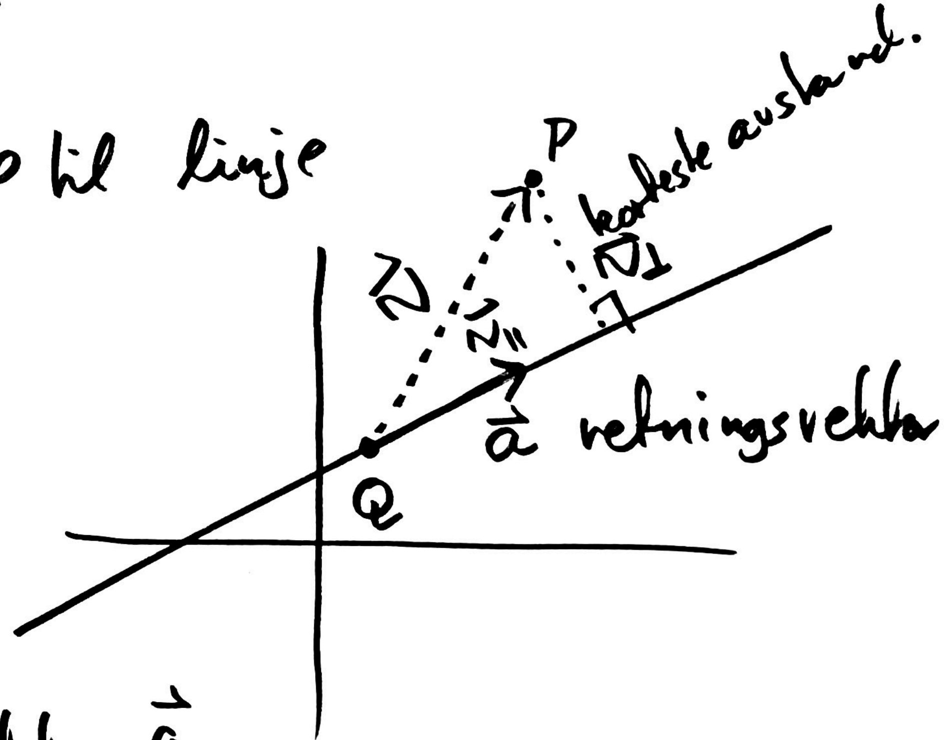
$$\vec{v}_{\perp} = \frac{1}{13}[32, 48] \approx [2.461, 3.692]$$

Korteste avstand fra punkt P til linje

er lik $|\vec{v}_{\perp}|$

hvor $\vec{v} = \vec{QP}$

$\vec{a}$ retningsvektor



ex

Linje gjennom origo m. retningsvektor $\vec{a}$

punkt P $\quad \vec{OP} = \vec{v}$

$P(5,2)$ og $\vec{a} = [-3,2]$

Korteste avstand $|\vec{v}_{\perp}|$

$$|\vec{v}_{\perp}| = \frac{16}{13}|[2,3]| = \frac{16}{\sqrt{13}}$$

14a) $\sqrt{3}\sin v + \cos v > 0$ $\qquad v \in [0, 2\pi]$

1) $\cos v = 0$ $\qquad \sin v = \pm 1$ $\qquad$ løsning for $\sin v = 1$: $\underline{v = \frac{\pi}{2}}$

$\cos v \neq 0$ $\qquad$ deler med $\cos v$:

2) $\cos v > 0$ $\qquad \sqrt{3}\tan v + 1 > 0 \Leftrightarrow \tan v > \frac{-1}{\sqrt{3}}$

3) $\cos v < 0$ $\qquad \sqrt{3}\tan v + 1 < 0 \Leftrightarrow \tan v < \frac{-1}{\sqrt{3}}$