17.01.23 Potensrekker

Geometriske rekker $S_n = 1 + x + x^2 + \cdots + x^n$

$n+1$ element

$$(1-x)S_n = 1 + x + x^2 + x^3 + \cdots + x^n - \left(x + x^2 + x^3 + \cdots + x^n + x^{n+1}\right)$$

$$= 1 - x^{n+1}$$

så $S_n = \begin{cases} \dfrac{1-x^{n+1}}{1-x} = \dfrac{x^{n+1}-1}{x-1} & x \neq 1 \\ n+1 & x = 1 \end{cases}$

$$S_n = \frac{1}{1-x} - \frac{x^{n+1}}{1-x}$$

$|x| < 1$ $\lim_{n\to\infty} S_n = \frac{1}{1-x}$ siden $x^n \to 0$.

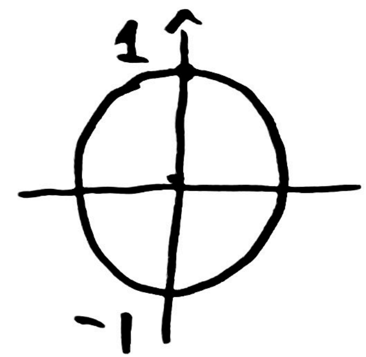
$$1+x+x^2+\cdots = \frac{1}{1-x} \qquad |x|<1$$

La $x = y^2$ $\quad 1+y^2+y^4+y^6+\cdots = \frac{1}{1-y^2} \qquad |y|<1$

$x = \sin y \qquad 1+\sin y+\sin^2 y+\sin^3 y+\cdots$

$y \in [0, 2\pi]$

For hvilke y konvergerer rekken og hva er da summen?



konverger hvis kvotienten: $|x| = |\sin y| < 1$

konvergerer: $\underline{y \neq \frac{\pi}{2} \text{ og } \frac{3\pi}{2}}$

summen er da $\frac{1}{1-x} = \frac{1}{1-\sin(y)}$

$$1 + (2-x) + (2-x)^2 + (2-x)^3 + \cdots$$

$y = (2-x)$ geometrisk rekke med kvotient y.

konvergerer når $-1 < y < 1$: $\underline{\underline{1 < x < 3}}$

$-1 < 2-x < 1$

$-1 < 2-x$: $x < 2+1 = 3$

$2-x < 1$: $1 = 2-1 < x$

summen : $\dfrac{1}{1-y} = \dfrac{1}{1-(2-x)} = \dfrac{1}{x-1} = \dfrac{-1}{1-x}$

oppg. Når konvergerer $1 + e^x + e^{2x} + e^{3x} + \cdots$

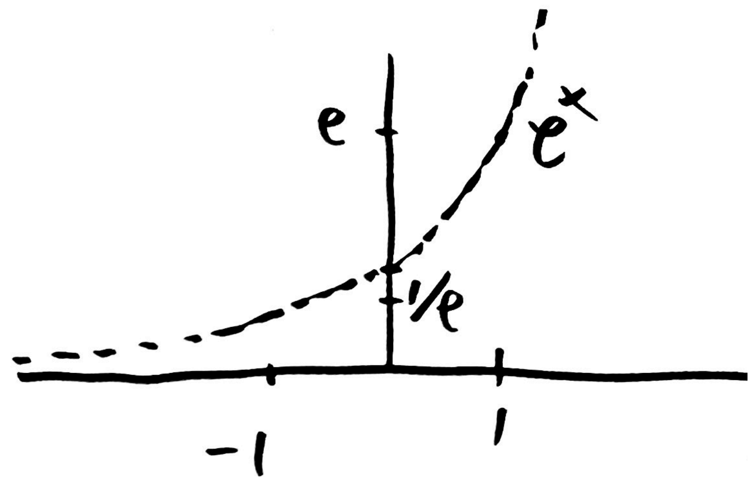
og hva blir da summen?

kvotienten er e^x

$e^x = |e^x| < 1$

konvergerer for alle $x < 0$.

summen $\underline{\dfrac{1}{1-e^x}}$



Eks

$$x^2 + x^5 + x^8 + x^{11} + \cdots$$ geometrisk rekke.

$$= x^2(1 + x^3 + x^6 + x^9 + \cdots)$$

$$= x^2 \cdot \frac{1}{1-x^3}$$ for $|x| < 1$

$(|x^3| < 1)$

divergerer for $|x| \geq 1$.

$$1+x+x^2+\cdots+x^n = \frac{1}{1-x} + \frac{-x^{n+1}}{1-x}$$

Deriverer begge sider m.h.t x

$$0+1+2x+3x^2+\cdots+nx^{n-1} = \frac{-1}{(1-x)^2}\underbrace{(1-x)'}_{-1} = \frac{1}{(1-x)^2} + \left(-\frac{(n+1)x^n(1-x)+x^{n+1}}{(1-x)^2}\right)$$

$$\left(-\frac{x^{n+1}}{1-x}\right)' = -\left(x^{n+1}\cdot\frac{1}{1-x}\right)' = -\left(\left(x^{n+1}\right)'\frac{1}{1-x} + x^{n+1}\left(\frac{1}{1-x}\right)'\right)$$

$$= -\underbrace{\left((n+1)x^n\frac{1}{1-x} + x^{n+1}\frac{1}{(1-x)^2}\right)}$$

Når $|x|<1$ vil leddet gå mot 0 når $n\to\infty$.
(siden $nx^n \to 0$)

$$1+2x+3x^2+\cdots = \sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

$|x| < 1$

divergerer når $|x| \geq 1$.

Potensrekker: grense av polynomer hvis vi tar med flere og flere ledd.

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots$$

$$\underline{\sum_{n=0}^{\infty} a_n x^n}.$$

Resultat $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$

$n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$

$0! = 1$

Rekken har samme derivert som e^x i $x = 0$

$$(e^x)'' = (e^x)' = e^x$$

$$(e^x)^{(n)}(0) = e^x\big|_{x=0} = 1.$$

Merk

$$(x^m)^{(n)} = m(m-1)(m-2)\cdots(m-n+1)x^{m-n}$$

$$(x^m)^{(m)}\big|_{x=0} = m!$$

	$\sin x$	$(\sin x)' = \cos x$	$(\sin x)'' = -\sin x$	$(\sin x)''' = -\cos x$
$x=0$	0	1	0	-1

$(\sin x)^{(4)} = \sin x$ tilbake igjen.

Taylor polynom til $f(x)$

$$P_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(0)}{i!} x^i$$

$$P_n^{(i)}(0) = f^{(i)}(0) \text{ for } i \leq n.$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$$

alle x

$n=0$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

e^{ix} $\quad i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$

$$e^{ix} = \sum_{n=0}^{\infty} \frac{(ix)^n}{n!}$$

$$= \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}}_{\cos x} + i \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}_{\sin x}$$

$$\underline{e^{ix} = \cos x + i \sin x}$$

oblig 7 3b) $S_n = \sum_{i=1}^{n} (-1)^i i$ Øving

$S_1 = -1$

$S_2 = -1 + 2 = 1$

$S_3 = -1 + 2 - 3 = -2$

$S_4 = -1 + 2 - 3 + 4 = 2$

$S_5 = $ — " —— $- 5 = -3$

$S_6 = $ —— " —— $+ 6 = 3$

Hva er S_n?