26 mars 25

Geometrisk rekke

$$a_1\overbrace{(1+k+k^2+\cdots k^{n-1})}^{n \text{ ledd}} = \begin{cases} a_1\left(\frac{1-k^n}{1-k}\right) & k \neq 1 \\ a_1 n & k = 1 \end{cases}$$

$a_{n+1} = k a_n$

$\uparrow$ kvotient

$a_n = k^{n-1} \cdot a_1$

$$a_1(1+k+k^2+\cdots) = a_1 \frac{1}{1-k}$$

$|k| < 1$

divergerer hvis $|k| \geq 1$

siffer

$$123 = 1 \cdot 10^2 + 2 \cdot 10 + 3$$

$$0.123 = \frac{1}{10} + \frac{2}{100} + \frac{3}{1000} = 1 \cdot 10^{-1} + 2 \cdot 10^{-2} + 3 \cdot 10^{-3}$$

$$a = 0.a_1 a_2 a_3 a_4 \ldots = \sum_{n=1}^{\infty} a_n \cdot 10^{-n}$$ konvergerer.

$\uparrow$ fjerde desimal

$0 \leq a_n \leq 9$ heltall.

n-te delsum av a er

$$\sum_{i=1}^{n} a_i 10^{-i} = \frac{a_1 a_2 a_3 \ldots a_n}{10^n}$$

er rasjonale tall.

Gjentagende desimaler

$0.333\ldots = 0.\underline{3}$

$0.35444\ldots = 0.35\underline{4}$

$0.585858\ldots = 0.\underline{58}$

$45.7877877877\ldots = 45.7\underline{877}$ det er også lik $= 45.\underline{787}$

$134.3434\ldots = 134.\underline{34}$

$$0.\underline{34} = 0.\underbrace{34}\underbrace{34}\underbrace{34}\ldots$$

$$= \sum_{n=1}^{\infty} 34 \cdot (100)^{-n}$$

$$= \frac{34}{100}\left(1 + \frac{1}{100} + \frac{1}{100^2} + \cdots\right)$$

geometrisk rekke.

$$= \frac{34}{100} \cdot \frac{1}{1-\frac{1}{100}} = \frac{34}{100-1} = \underline{\underline{\frac{34}{99}}}$$

$$0.585858\ldots = 0.\underline{58}$$

$$= \frac{58}{100}\left(1 + \frac{1}{100} + \frac{1}{100^2} + \cdots\right)$$

$$= \underline{\underline{\frac{58}{99}}}$$

+1

$$0.3747474\ldots = 0.3\underline{74}$$

$$= \frac{3}{10} + 0.0\underline{74} = \frac{3}{10} + \frac{74}{1000}\left(1 + \frac{1}{100} + \frac{1}{100^2} + \cdots\right)$$

$$= \frac{3}{10} + \frac{74}{1000}\,\frac{100}{99} = \frac{3}{10} + \frac{74}{990}$$

$$= \frac{3 \cdot 99}{990} + \frac{74}{990} = \underline{\frac{371}{990}}$$

$$0.\underline{a_1 \ldots a_n} = \frac{a_1 a_2 \cdots a_n}{10^n}\left(1 + 10^{-n} + 10^{-2n} + \cdots\right.$$

$$= \frac{a_1 \cdots a_n}{10^n}\,\frac{1}{1 - 10^{-n}} = \frac{a_1 \cdots\cdot a_n}{10^n - 1}$$

$$= \frac{a_1 a_2 \cdots\cdot a_n}{\underbrace{999\ldots9}_{n \text{ siffer}}}$$

Alle desimaltall med gjentagende siffre er lik et rasjonalt tall.

+ /

Alle rasjonale tall har gjentagende siffer i sin desimalekspansjon.

$$\frac{11}{8} = \frac{8+3}{8} = 1 + \frac{3}{8} = 1 + \frac{1}{10} \cdot \frac{3 \cdot 10}{8}$$

$$= 1 + \frac{1}{10} \cdot \left(\frac{3 \cdot 8 + 6}{8} \right)$$

$$= 1 + \frac{1}{10}\left(3 + \frac{3}{4}\right)$$

$$= 1 + \frac{1}{10}\left(3 + \frac{1}{10} \frac{30}{4}\right)$$

$$= 1 + \frac{3}{10} + \frac{1}{100}\left(7 + \frac{2}{4}\right)$$

$$= 1 + \frac{3}{10} + \frac{7}{100} + \frac{1}{100}\left(\frac{10}{2}\right) \cdot \frac{1}{10}$$

$$= 1 + \frac{3}{10} + \frac{7}{100} + \frac{5}{1000}$$

$$= \underline{1.375}$$

+1

$$\frac{1}{3} = \frac{1}{10}\left(\frac{10}{3}\right) = \frac{1}{10}\left(3 + \frac{1}{3}\right) = \frac{3}{10} + \frac{1}{10}\left(\frac{1}{10}\,\frac{30}{3}\right)$$

$$= \frac{3}{10} + \frac{1}{100}\left(3 + \frac{1}{3}\right) \quad \text{etc}$$

$$= 0.3333\ldots$$

$$\frac{1}{7} = \frac{1}{10}\left(\frac{10}{7}\right) = \frac{1}{10}\left(1 + \frac{3}{7}\right) = \frac{1}{10} + \frac{1}{100}\left(\frac{30}{7}\right)$$

$$= \frac{1}{10} + \frac{1}{100}\left(4 + \frac{2}{7}\right) = 0.14 + \frac{1}{1000}\left(\frac{20}{7}\right) = 0.14 + 10^{-3}\left(2 + \frac{6}{7}\right)$$

$$= 0.142 + 10^{-4}\frac{60}{7} = 0.142 + \frac{7\cdot 8 + 4}{7}\cdot 10^{-4} = 0.1428 + 10^{-5}\frac{40}{7}$$

$$= 0.1428 + 10^{-5}\left(5 + \frac{5}{7}\right) = 0.14285 + 10^{-6}\underbrace{\frac{50}{7}}_{\frac{49+1}{7}}$$

$$= 0.142857 + 10^{-6}\cdot\frac{1}{7}$$

$$= 0.\underline{142857}$$

(Restene som forekommer
1, 3, 2, 6, 4, 5)

$\frac{m}{n}$ har tilsvarende en desimal ekspansion med
$n-1$ eller færre siffer som gjentas.

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Eksamen mai 24

$a_1 = \frac{1}{3} + \frac{5}{3}(n-1)$

minste n slik at $a_n \geq 200$

$\frac{1}{3} + \frac{5}{3}(n-1) \geq 200$ $\quad 1 \cdot 3$

$1 + 5(n-1) \geq 600$

$5n \geq 604$

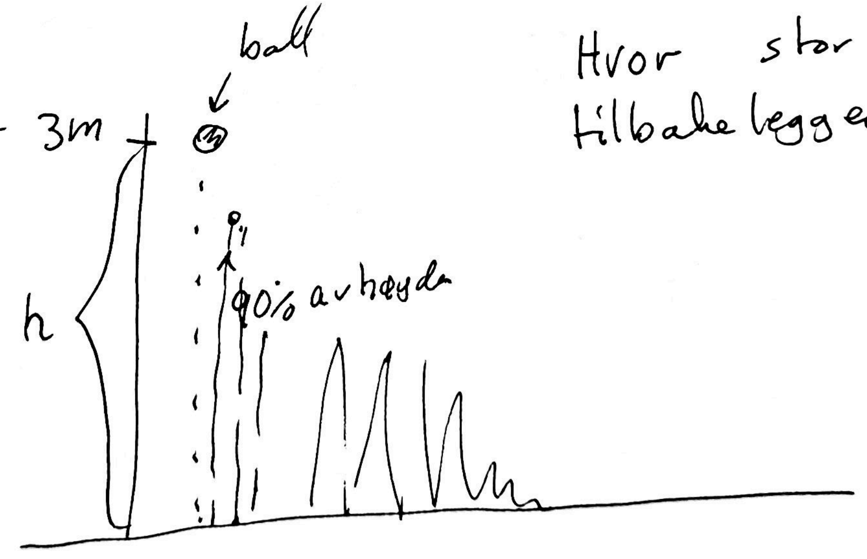
$n \geq \frac{604}{5} = \frac{60 \cdot 10}{5} + \frac{4}{5}$

$\geq 120 + \frac{4}{5}.$

$n = 121$ er minste n slik at $a_n \geq 200.$

$a_{121} = \frac{1}{3} + \frac{5}{3}(120) = \underline{200 + \frac{1}{3}}$

Eksamen mai 19 6 b) variant 3m



Hvor stor distanse ? Tilbake legger ballen?

$k = 90\% = \frac{90}{100}$
$= 0.9$

$$D = h + \underbrace{0.9}_{k} \cdot h \cdot 2 + k^2 \cdot h \cdot 2 + k^3 \cdot h \cdot 2 + \cdots$$

$$= -h + 2h\Big(\underbrace{1 + k + k^2 + k^3 + \cdots}_{\frac{1}{1-k}}\Big)$$

$$D = \frac{2h}{1-k} - h$$

$$= 3\left(\frac{2}{1-0.9} - 1\right) = 3 \cdot (20-1) = \underline{\underline{57}}\ m$$

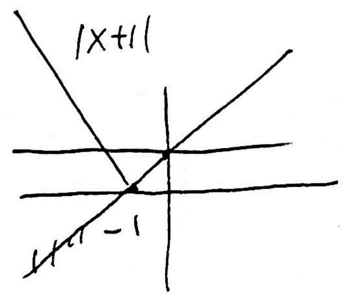
geometrisk rekke.

c) $(x+1) + 1 + \frac{1}{x+1} + \frac{1}{(x+1)^2} + \cdots$ kvotient $\frac{1}{x+1}$.

Konvergensintervallet. $\quad$ (Finn løsningsmengden)

Konvergerer når $\left|\frac{1}{x+1}\right| < 1$

$\Leftrightarrow \quad 1 < |x+1| \quad$ (og $x \neq -1$)



ser at løsningene er $\underline{\langle -\infty, -2\rangle \cup \langle 0, \infty\rangle}$

Vi finner også summen i dette tilfellet

$$(x+1)\left(1 + \frac{1}{x+1} + \frac{1}{(x+1)^2} + \cdots\right)$$

$$= (x+1)\,\frac{1}{1-\frac{1}{x+1}} = \frac{1}{x+1-1} = \underline{\underline{\frac{1}{x}}}$$

Eks. august 24

$$a_1\left(1+k+\cdots k^{29}\right) \qquad a_1 = 1$$

$$= a_1 \cdot \frac{k^{30}-1}{k-1} \qquad k = 1.2$$

$$= 1 \cdot \frac{(1.2)^{30}-1}{0.2} = 5\left((1.2)^{30}-1\right)$$

$$\approx \underline{1186.9}$$

Summenotasjon $\quad \Sigma \quad$ (sigma) $\quad \sigma$

$$\sum_{k=3}^{5} a_k = a_3 + a_4 + a_5$$

Oppg skriv

a) $a_{-3} + a_{-2} + \cdots + a_{11}$ med Σ-notasjon

b) $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16}$ —"—

a) $\sum_{k=-3}^{n} a_k$

b) $\sum_{n=0}^{5} a_{1+3n} = \sum_{k=1}^{6} a_{3k-2}$

Skriv ut summen $\sum_{i=-1}^{4} i^2$

$(-1)^2 + 0^2 + 1^2 + 2^2 + 3^2 + 4^2$

$1 + 0 + 1 + 4 + 9 + 16$

oppg. $\sum_{k=1}^{\infty} x^k = 5$ Løs for x.

$(x + x^2 + x^3 + \cdots)$

$x(1 + x + x^2 + \cdots) = x \cdot \frac{1}{1-x}$ når $|x| < 1$.

$\frac{x}{1-x} = 5 \iff x = 5(1-x) = 5 - 5x$

$6x = 5$

$|x| < 1$

Løsningen er $\underline{\underline{x = 5/6}}$

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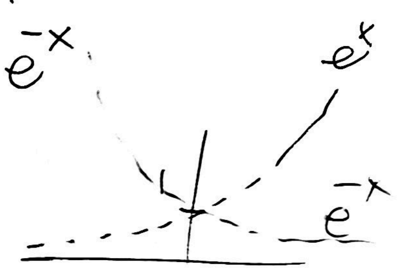
Kont. eks 23.

$S(x) = 1 + e^{-x} + e^{-2x} + \cdots$

a) Når konvergerer rekken.

b) Bestem x slik at $S(x) = 3$.

$S(x) = \sum_{n=0}^{\infty} e^{-nx} = \sum_{n=0}^{\infty} (e^{-x})^n$. geometrisk rekke kvotienten er e^{-x}.



$|e^{-x}| = e^{-x} < 1$

a) konvergerer når $\underline{x > 0}$

b) Summen er $\frac{1}{1-e^{-x}}$ når $x > 0$

$\frac{1}{1-e^{-x}} = 3 \iff 1 - e^{-x} = \frac{1}{3}$

$e^{-x} = 1 - \frac{1}{3} = \frac{2}{3}$

$\underline{e^{x} = 3/2}$

$x = \underline{\ln(3/2)}$

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Kont. eksamen 2018

4 d) – f) Tablett 1.5mg virkestoff. Tas daglig
Brytes ned til 80% per dag.

Mengden virkestoff i kroppen er :

d) $1.5 + 1.5 \cdot 0.8 + 1.5 \cdot 0.8^2 + \cdots + 1.5 \cdot (0.8)^{n-1}$ (når n tabletter er tatt)

nå | i går | i forgårs | etc

e) kvotienten i den geometriske rekken er $k = 0.8$

$a_n = 1.5 \cdot (0.8)^{n-1}$ $\qquad$ $\underline{a_6 = 1.5 \cdot (0.8)^5}$

Delsummen $S_6 = 1.5 \dfrac{(0.8)^6 - 1}{0.8 - 1} = \underline{7.5(1 - 0.8^6)}$

f) $S_n = 7.5(1 - (0.8)^n) < 7.5$ for alle n.

Mengden virkestoff i kroppen vil ikke overstride $\underline{7.5\,mg}$

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