7 april 25

Ubestemte integraler

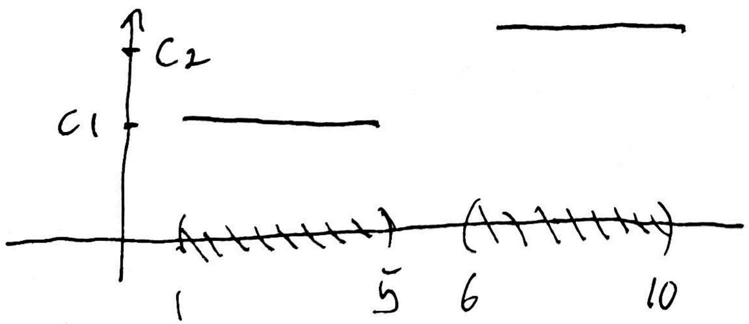
15D

$$\underset{\text{integral-tegn}}{\int} \underset{\text{integrand}}{f(x)}\, \underset{\text{differensial.}}{dx} = \text{mengden av alle antideriverte til } f(x).$$

$(x^2)' = 2x$, $\quad (x^2+13)' = (x^2)' + (13)' = 2x$

Hvis $f(x)$ og $g(x)$ har samme derivert $(f(x) - g(x))' = 0$ (for alle x i def. mengden),

da må $(f(x) - g(x))$ være konstant på hver komponent av (den felles) definisjonsmengden.



Hvis $f'(x) = g'(x)$ for alle x, da er $f(x) = g(x) + C$

↑ konstant

konstanten kan variere mellom komponentene.

$F(x)$ være en antiderivert til $f(x)$, da er

$$\int f(x)\,dx = F(x) + C$$

$(e^x)' = e^x$ så $\int e^x\,dx = e^x + C$

$(\sin x)' = \cos x$ så $\int \cos x\,dx = \sin x + C$

$(-\cos x)' = \sin x$ så $\int \sin x\,dx = -\cos x + C$

$(x^n)' = nx^{n-1}$

$(x^{n+1})' = (n+1)x^n$ hvis $n \neq -1$ så $\left(\frac{1}{n+1} x^{n+1}\right)' = x^n$

$$\underline{\int x^n \, dx = \frac{x^{n+1}}{n+1} + C} \qquad n \neq -1$$

$(\ln x)' = \frac{1}{x}$ $\quad x > 0$

$(\ln(-x))' = \frac{(-x)'}{-x} = \frac{-1}{-x} = \frac{1}{x}$ $\quad x < 0$

$|x| = \begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$

$(\ln|x|)' = \frac{1}{x}$ $\quad x \neq 0$

så

$\int \frac{1}{x} \, dx = \ln|x| + C$ (en konstant for $x < 0$ og en annen for $x > 0$)

$$f(x) = \begin{cases} \ln(x) + 3 & x > 0 \\ \ln(-x) + 7 & x < 0 \end{cases} \qquad D_f = \mathbb{R} \setminus \{0\}$$

$$= \begin{cases} \ln|x| + 3 & x > 0 \\ \ln|x| + 7 & x < 0 \end{cases}$$

$$f'(x) = \frac{1}{x} \qquad x \neq 0.$$

Ubestemte integraler er lineære.

$(f+g)' = f' + g'$

Gitt: $F'(x) = f(x)$ og $G'(x) = g(x)$

$(F+G)'(x) = F'(x) + G'(x)$

$= f(x) + g(x)$

$$\boxed{\int f + g \, dx = \int f \, dx + \int g \, dx}$$

$$\int 2x + 3x^2 \, dx = (x^2 + x^3) + C$$

$$\int 2x\,dx + \int 3x^2\,dx$$

$$x^2 + C_1 + x^3 + C_2 = (x^2 + x^3) + \overbrace{C_1 + C_2}^{C}$$

k konstant

$$(k \cdot f(x))' = k \cdot f'(x)$$

$$F'(x) = f(x)$$

$$(k \cdot F(x))' = k\,F'(x) = k\,f(x)$$

$$\boxed{\int k\,f(x)\,dx = k \cdot \int f(x)\,dx}$$

$$\int 13x\,dx = 13 \int x\,dx = 13\left(\frac{x^2}{2} + c\right)$$

$$= \frac{13}{2} x^2 + 13c$$

$$\int 0 \cdot x \, dx = 0 \int x \, dx = 0\left(\frac{x^2}{2} + C \right) = 0$$

hvor er konstanleddet?

Tenk heller

$$\int 0 \cdot x \, dx = \underbrace{0 \int x \, dx}_{\text{ønsker én antiderive}} + C = \underline{0 + C} = C$$

$$= 0 \cdot \frac{x^2}{2} + C$$

$$= 0 + C$$

$$\int 0 \cdot x \, dx = \int 0 \, dx = C$$

eks

$$\int -3x^2 + \frac{4}{x^2} \, dx = \int -3x^2 \, dx + 4 \int x^{-2} \, dx$$

$$= -x^3 + 4 \frac{x^{-1}}{-1} + C$$

$$= \underline{-x^3 - \frac{4}{x} + C}$$

opg

$$\int \sqrt{x} - \frac{3}{x} \, dx$$

$$= \int x^{1/2} dx + (-3)\int \frac{1}{x} dx$$

$$= \frac{x^{\frac{1}{2}+1}}{3/2} + (-3)\ln|x| + C = \underline{\frac{2x\sqrt{x}}{3} - 3\ln|x| + C}$$

opg

$$\int x^2 \sqrt[3]{8x} + (3x)^2 (x-4) \, dx$$

$$= \int x^2 \sqrt[3]{8}\, x^{1/3} dx + \int 3^2 x^2 (x-4) \, dx$$

$$= \int 2x^{7/3} dx + 9\int x^3 - 4x^2 \, dx$$

$$= \frac{2x^{10/3}}{10/3} + 9\left(\frac{x^4}{4} - \frac{4x^3}{3}\right) + C$$

$$= \underline{\frac{3}{5} x^3 \sqrt[3]{x} + \frac{9}{4} x^4 - 12x^3 + C}$$

$$\int 3x + (3-x)^2 x \; dx$$

$$= \int 3x\,dx + \int 9x - 6x^2 + x^3\,dx$$

$$= \int 12x\,dx + \int -6x^2\,dx + \int x^3\,dx$$

$$= 12\frac{x^2}{2} - 6\frac{x^3}{3} + \frac{x^4}{4} + C$$

$$= \underline{6x^2 - 2x^3 + \frac{1}{4}x^4 + C}$$

$$\int (2+x)^{17}\,dx$$

$$= \underline{\frac{1}{18}(2+x)^{18} + C}$$

$$\left((2+x)^n\right)' = n(2+x)^{n-1} \cdot \overbrace{(2+x)'}^{1}$$

$$= n(2+x)^{n-1}.$$

så $\left(\frac{1}{18}(2+x)^{18}\right)' = (2+x)^{17}.$

$$\int e^{-4x}\,dx = \underline{\underline{\frac{-1}{4} e^{-4x} + C}}$$

$$(e^{-4x})' = e^{-4x}(-4x)' = -4e^{-4x}$$

$$\left(\frac{-1}{4} e^{-4x}\right)' = e^{-4x}$$

$$F'(x) = f(x)$$

$$(F(ax+b))' = F'(ax+b) \cdot \overbrace{(ax+b)'}^{a}$$

$$\left(\frac{1}{a} F(ax+b)\right)' = F'(ax+b) = f(ax+b)$$

$u = ax+b$

$$\int f(ax+b)\,dx = \frac{1}{a}\int f(u)\,du$$

Linear substitusjon (variabelbytte)

$$\int \sin(\underbrace{5x+2}_{u})\,dx = \frac{1}{5}\int \sin(u)\,du$$

$$= \frac{1}{5}\left(-\cos(u)\right) + c$$

$$= \frac{-1}{5}\cos(5x+2) + C$$

$$u' = 5$$

$$\frac{du}{dx} = 5$$

$$du = 5dx$$

$$\frac{1}{5}du = dx$$

$$\int x(3x+1)^{12}\,dx$$

$$u = 3x+1 \qquad du = 3dx$$

$$u-1 = 3x$$

$$x = \frac{1}{3}(u-1)$$

$$\int \frac{1}{3}(u-1)\,u^{12}\,\frac{1}{3}du$$

$$= \frac{1}{9}\int u^{13} - u^{12}\,du = \frac{1}{9}\left(\frac{u^{14}}{14} - \frac{u^{13}}{13}\right) + C$$

$$= \frac{(3x+1)^{13}}{9}\left(\frac{3x+1}{14} - \frac{1}{13}\right) + C$$

oppg. a) $\int \frac{1}{x-2}\,dx$ $\qquad u = x-2$, $du = dx$

$= \int \frac{1}{u}\,du = \ln|u| + C = \underline{\ln|x-2| + C}$

b) $\int \frac{1}{e^{2x+1}}\,dx = \int (e^{2x+1})^{-1}\,dx = \int e^{-2x-1}\,dx$ $\quad \Big| \; u = -2x-1$, $du = -2\,dx$

$= \int e^{u}\left(\frac{-1}{2}\right)du = \frac{-1}{2}e^{u} + C = \frac{-1}{2}e^{-2x-1} + C$

$= \underline{\frac{-1}{2e^{2x+1}} + C}$

c) $\int \sqrt{4-3t}\,dt$

Lar $u = 4-3t$ $\qquad du = -3\,dt$ så $dt = \frac{-1}{3}\,du$

$= \int \sqrt{u}\left(\frac{-1}{3}\right)du = \frac{-1}{3}\int u^{1/2}\,du$

$= \frac{-1}{3}\,\frac{u^{3/2}}{3/2} + C = \frac{-2}{9}(4-3t)^{3/2} + C$

$= \underline{\frac{-2}{9}(4-3t)\sqrt{4-3t} + C}$

$$\int \sin^2 x \, dx$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$ doBling av vinkel

$$\sin^2 x + \cos^2 x = 1$$ Pytagoras

$$\cos 2x = (1 - \sin^2 x) - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$\int \sin^2 x \, dx = \int \frac{1}{2} - \frac{1}{2}\cos(2x) \, dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos(2x) \, dx$$

Linear substitusjon $u = 2x$ $du = 2dx$

$$= \frac{x}{2} - \frac{1}{2}\int \cos(u) \frac{1}{2} du$$

$$= \frac{x}{2} - \frac{1}{4}\int \cos(u) \, du = \frac{x}{2} - \frac{1}{4}\sin(u) + C$$

$$\int \sin^2 x = \underline{\frac{x}{2} - \frac{1}{4}\sin(2x) + C}$$

$$\int \frac{4}{(5-7x)^3}\,dx$$

$$u = 5-7x$$
$$du = -7\,dx$$
$$\frac{-1}{7}du = dx$$

$$= \int 4u^{-3}\left(\frac{-1}{7}\right)du$$
$$= \frac{-4}{7}\int u^{-3}\,du = \frac{-4}{7}\,\frac{u^{-2}}{-2} + C$$
$$= \frac{2}{7}(5-7x)^{-2} + C = \frac{2}{7(5-7x)^2} + C$$

ØVING

$$\int u'(x)\, f(u(x))\, dx = \int f(u)\, du$$ substitusjon

$$F'(u) = f(u)$$

$$\frac{d}{dx}\left(F(u(x))\right) = F'(u(x)) \cdot u'(x)$$ kjerneregel

$$= f(u(x))\, u'(x)$$

$u = 4 + x^3$

$du = 3x^2\, dx$

$u' = 3x^2$

$\frac{1}{3}u' = x^2$

$$\int \underbrace{x^2}_{\frac{1}{3}u'} \underbrace{(4+x^3)^7}_{u^7}\, dx$$

$$= \frac{1}{3} \int u^7\, du = \frac{1}{3} \cdot \frac{u^8}{8} + C$$

$$= \frac{(4+x^3)^8}{24} + C$$

$$\int_a^b u'(x)\, f(u(x))\, dx = F(u(x))\Big|_a^b = F(u(b)) - F(u(a))$$

$$= \int_{u(a)}^{u(b)} f(u)\, du$$ substitusjon og bestemte integral.