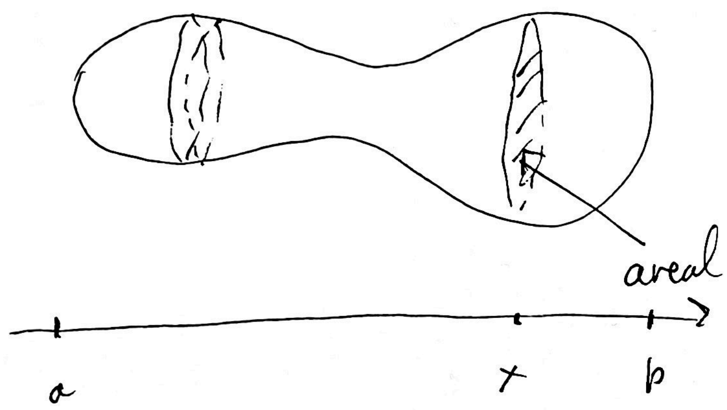
23 april 25

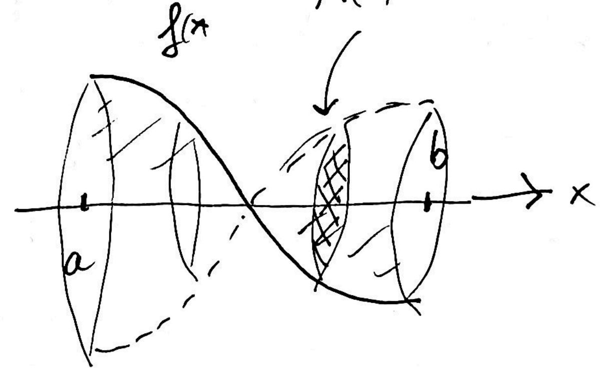
15 F Omdreiningslegemer



$$V = \int_a^b A(x)\,dx$$

areal $A(x)$, tverrsnittarealet.

omdreiningslegemer



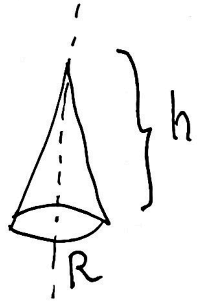
$A(x) = \pi |f(x)|^2 = \underline{\pi f(x)^2}$

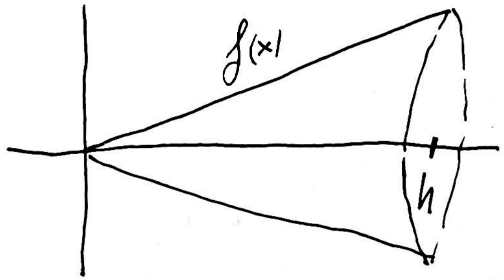
Roterer området avgrenset av grafen til f roteres om x-aksen.

Volumet er $V = \int_a^b A(x)\,dx = \int_a^b \pi f(x)^2\,dx$

$= \underline{\pi \int_a^b f(x)^2\,dx}$

Kjegle





$f(0) = 0$

$f(h) = R$

$f(x)$ lineær

$f(x) = x \cdot \frac{R}{h} = \underline{\frac{R}{h} x}$

Volumet til kjeglen

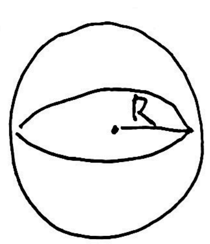
$$V = \pi \int_0^h \left(\frac{R}{h} x\right)^2 dx$$

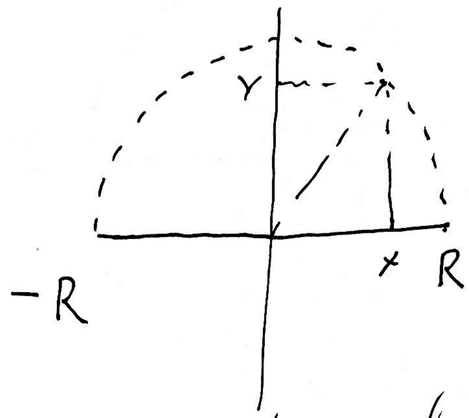
$$= \pi \frac{R^2}{h^2} \int_0^h x^2 dx = \frac{\pi R^2}{h^2} \underbrace{\frac{x^3}{3}\Big|_0^h}_{\frac{h^3}{3} - \frac{0^3}{3}}$$

$$V = \frac{\pi R^2}{h^2} \cdot \frac{h^3}{3}$$

$$= \underline{\frac{\pi}{3} R^2 h}$$

Kule





$x^2+y^2=R^2$

$y>0$

$y=\sqrt{R^2-x^2}$

Rotasjonslegemet er en kule med radius R.

Volum kule $V=\int_{-R}^{R}\pi y(x)^2\,dx = \pi\int_{-R}^{R} R^2-x^2\,dx$

$\left(\text{benytta: } \int_{-R}^{0}\pi y^2\,dx = \int_{0}^{R}\pi y^2\,dx\right)$

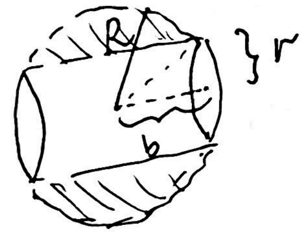
$$V = 2\pi\int_0^R R^2-x^2\,dx = 2\pi\left(R^2\cdot x-\frac{x^3}{3}\right)\Big|_0^R$$

$$= 2\pi\left(\underbrace{R^2\cdot R-\frac{R^3}{3}}_{\frac{2R^3}{3}}-0\right)$$

$$\underline{V=\frac{4\pi R^3}{3}}$$

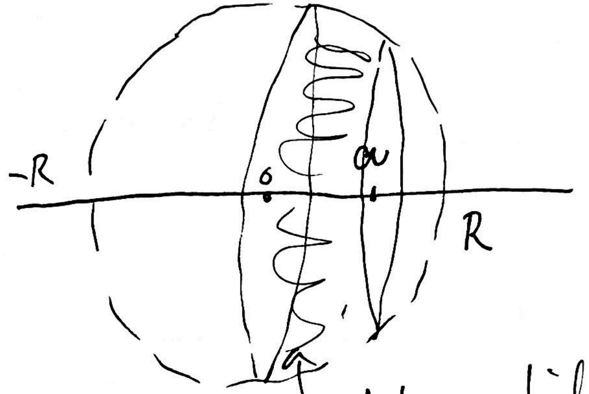
Oppgave

Volum til en perle



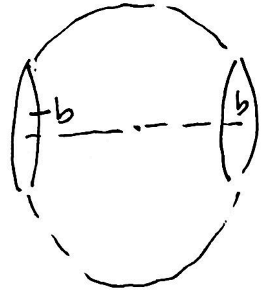
$$b^2 + r^2 = R^2$$
$$\underline{b = \sqrt{R^2 - r^2}}$$

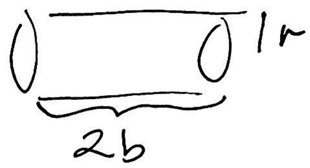
En sylinder bores ut
av en kule med radius R
sylinderen har radius r
og går rett gjennom kulen



Volum til denne delen

$$V_a = \int_0^a \pi (R^2 - x^2)\,dx$$
$$\pi \left(R^2x - x^3/3\right)_0^a$$
$$V_a = \pi \left(R^2 a - a^3/3\right)$$
$$= \underline{\pi a \left(R^2 - \frac{a^2}{3}\right)}$$

$V_{perle} = V_{dur}$ 

$- \text{Volum sylinder}$ 

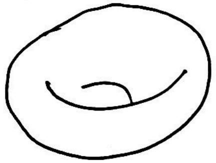
$$V_{perle} = 2\pi b\left(R^2 - \frac{b^2}{3}\right) - 2b \cdot \pi r^2$$

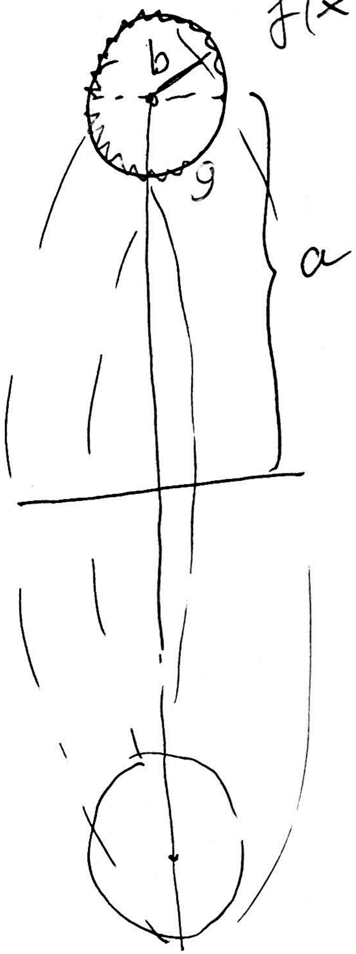
$$= 2\pi\sqrt{R^2-r^2}\underbrace{\left(R^2 - \frac{R^2-r^2}{3}\right)} - 2\pi r^2\sqrt{R^2-r^2}$$

$$= 2\pi\sqrt{} \cdot \left(\frac{2R^2}{3} + \frac{r^2}{3}\right) - 2\pi\sqrt{} \cdot r^2$$

$$= 2\pi\sqrt{R^2-r^2}\left(\frac{2R^2}{3} - \frac{2r^2}{3}\right) = \underline{\frac{4\pi}{3}\sqrt{R^2-r^2}\left(R^2-r^2\right)}$$

Torus





$f(x) = a + \sqrt{b^2 - x^2}$

$g(x) = a - \sqrt{b^2 - x^2}$

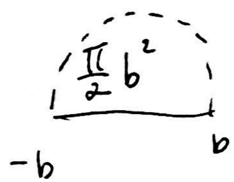
$b < a$

$$V_{torus} = \int_{-b}^{b} \pi f^2(x)\,dx - \int_{-b}^{b} \pi g^2(x)\,dx$$

$$= \pi \int_{-b}^{b} \underbrace{f^2(x) - g^2(x)}_{(f(x)+g(x))(f(x)-g(x))}\,dx$$

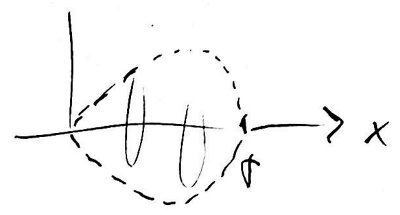
$$V_{torus} = \pi \int_{-b}^{b} 2a \cdot 2\sqrt{b^2 - x^2}\,dx$$

$$= 4\pi a \underbrace{\int_{-b}^{b} \sqrt{b^2 - x^2}\,dx}_{\frac{\pi}{2} b^2}$$



$$V_{torus} = \underline{2\pi^2 a b^2} = \underline{2\pi a \cdot \pi b^2}$$

$y(x) = \sqrt{x \sin x}$ $\qquad x \in [0, \pi]$



Finn volum til rotasjonslegemet.

$$V = \int_0^{\pi} \pi \, y(x)^2 \, dx$$

$$= \pi \int_0^{\pi} x \sin x \, dx$$

prøver med delvis integrasjon.

$$\int \underbrace{x}_{u} \, \underbrace{\sin x}_{v'} \, dx = u \cdot v - \int u' \cdot v \, dx$$

$v' = \sin x$

$v = -\cos x$

$$= x(-\cos x) - \int 1 \cdot (-\cos x) \, dx$$

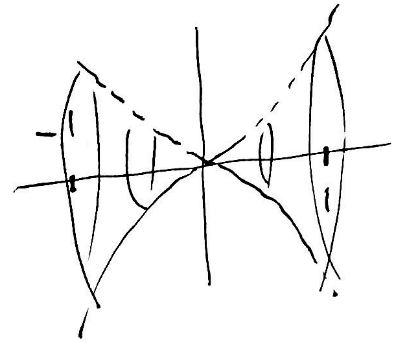
$$= -x \cos x + \int \cos x \, dx = \underline{-x \cos x + \sin x + C}$$

$$V = \pi \left(-x \cos x + \sin x \right) \Big|_0^{\pi} = \pi \left(-\pi \overbrace{\cos(\pi)}^{-1} + \overbrace{\sin \pi}^{0} - (0 + \sin(0)) \right)$$

$$V = \underline{\underline{\pi^2}}$$

(Tilsvarende ex oppg. 21')

$$Y(x) = \sinh(x) = \frac{e^{x} - e^{-x}}{2}$$



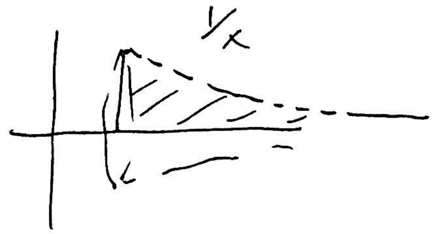
Området avgrenset av $Y(x)$ fra $x=-1$ til $x=1$ roteres om x-aksen.
Finn volumet til rotasjonslegemet som fremkommer ved rotasjon rundt x-aksen.

$$Y_{(x)}^2 = \left(\frac{e^{x} - e^{-x}}{2}\right)^2 = \frac{1}{4}\left(e^{2x} - 2 + e^{-2x}\right)$$

$\sinh(x)$ odde funksjon
$Y^2(x) = Y^2(-x)$

$$V = \int_{-1}^{1} \pi Y_{(x)}^2 \, dx$$
$$= \int_{-1}^{1} \frac{\pi}{4}\left(e^{2x} - 2 + e^{-2x}\right) dx$$
$$= 2\int_{0}^{1} \frac{\pi}{4}\left(e^{2x} - 2 + e^{-2x}\right) dx$$
$$= \frac{\pi}{2}\left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2}\right]_0^1$$
$$= \frac{\pi}{2}\left[\left(\frac{e^2}{2} - 2 - \frac{e^{-2}}{2}\right) - \left(\frac{1}{2} - 0 - \frac{1}{2}\right)\right]$$
$$V = \underline{\frac{\pi}{4}\left(e^2 - 2 - \frac{1}{e^2}\right)}$$

$f(x) = \frac{1}{x} \qquad x \in [1, \infty\rangle$



$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{n\to\infty} \int_1^n \frac{1}{x}\,dx$$

$$= \lim_{n\to\infty} \ln(n) = \infty$$

uendelig areal.

Omdreiningslegemet har derimot endelig volum

$$\int_1^{\infty} \pi \cdot \left(\frac{1}{x}\right)^2 dx = \lim_{n\to\infty} \pi \int_1^n \frac{1}{x^2}\,dx$$

$$= \lim_{n\to\infty} \pi \frac{-1}{x}\Big|_1^n = \lim_{n\to\infty} \pi\left(1 - \frac{1}{n}\right)$$

$$= \underline{\underline{\pi}}$$

+1)