7 sep
26

Ulikheter

Boka 44

$a < b$ a (ekte) mindre enn b $\Leftrightarrow$ $b > a$ b er (ekte) større enn a

$a \leq b$ a er mindre enn eller lik b
$\Updownarrow$
$b \geq a$ b er større enn eller lik a

($\leqslant$ skrives også $\leq$)

$a < b \Leftrightarrow a + c < b + c$ alle a, b og c

$a < b \Leftrightarrow 0 < b - a$ (legger vi til $-a$)
$\Leftrightarrow a - b < 0$ (—— $-b$)

$a < b \Leftrightarrow -b < -a$ (—— $-a-b$)
$\Leftrightarrow -a > -b$

ganger vi begge sider med -1 så blir ulikhet snudd.

$a < b \iff a - b < 0$

Ganger med $c > 0$: $ac - bc < 0 \iff ac < bc$

ulikheten bevares

Ganger med $c < 0$: $(a-b) \cdot c = ac - bc > 0$

produktet av to negative tall er positivt.

$\iff ac > bc$

↑

Ulikheten er snudd.

($c = 0$ $\quad ac = bc = 0$)

$2 < 3$ ganger med 4

$2 \cdot 4 < 3 \cdot 4$

$8 < 12$

$2 < 3$ ganger med -3 : $\quad 2(-3) > 3(-3)$

$-6 > -9$

$-a < -b$ gange med -1 : $a > b$ (snu!)

legg a og b til begge sider av ulikheten

$b < a \iff a > b.$

$x > 2$ mengden av x som gjør dette sant er

$\langle 2, \infty \rangle$

delmengden av $\mathbb{R}$

$2 \leq x < 5 \iff 2 \leq x$ og $x < 5$

$[2, 5\rangle$

$x^2 \leq 0 \iff x = 0$

$\{0\}$

Ulikheter er påstander (med variabler) som benytter ulikheter istedenfor likheter.

$$2x - 7 \geq 0$$

Løsningene til en ulikhet er mengden av verdiene til variablene som gjør påstanden sann.

$2x - 7 \geq 0$ legger til 7

$2x \geq 7$ gange med $1/2$ (dele med 2)

$x \geq \frac{7}{2}$

Løsningene er alle x slik at $x \geq 7/2$.

Løsningsmengden er $[7/2, \infty)$

$\frac{x}{2} + 1 < 2x + 4$ Lineær ulikhet

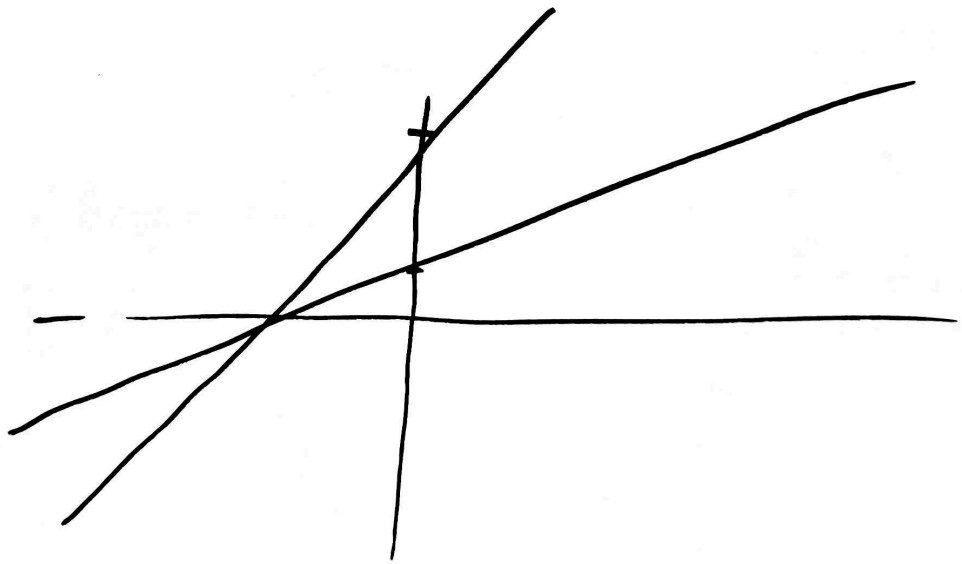
$\frac{x}{2} - 2x < 4 - 1$

$x\left(\frac{1}{2} - \frac{4}{2}\right) < 3 \iff \frac{-3}{2}x < 3$ ganger med $\frac{-2}{3}$

$\iff x > 3\left(\frac{-2}{3}\right)$

$\underline{x > -2}$

Løsningsmengden $\underline{\langle -2, \infty \rangle}$



Doble ulikhete $\quad \frac{x}{2} - 3 \leq x < 10 - x$

$\Leftrightarrow \frac{x}{2} - 3 \leq x \quad$ og $\quad x < 10 - x$

(begge skal være oppfylt)

I $\quad \frac{x}{2} - 3 \leq x \quad \Leftrightarrow \quad -3 \leq x - \frac{x}{2}$

$-3 \leq \frac{1}{2}x$

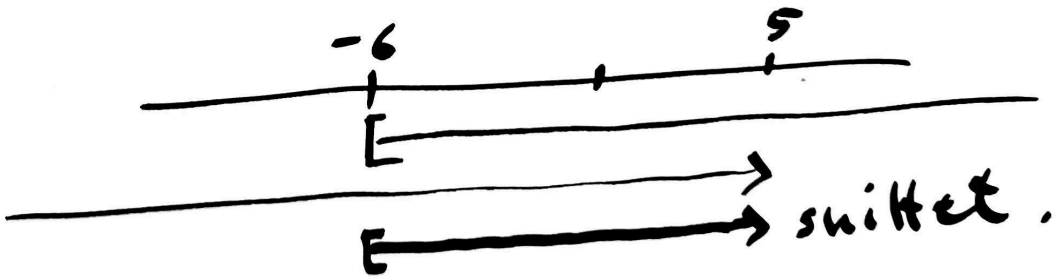
$-6 \leq x \qquad [-6, \infty\rangle$

II $\quad x < 10 - x \quad \Leftrightarrow \quad 2x < 10$

$x < 5 \qquad \langle -\infty, 5\rangle$

Felles løsningsmengde (begge påstandene er sanne)

$[-6, \infty\rangle \cap \langle -\infty, 5\rangle = \underline{[-6, 5\rangle}$



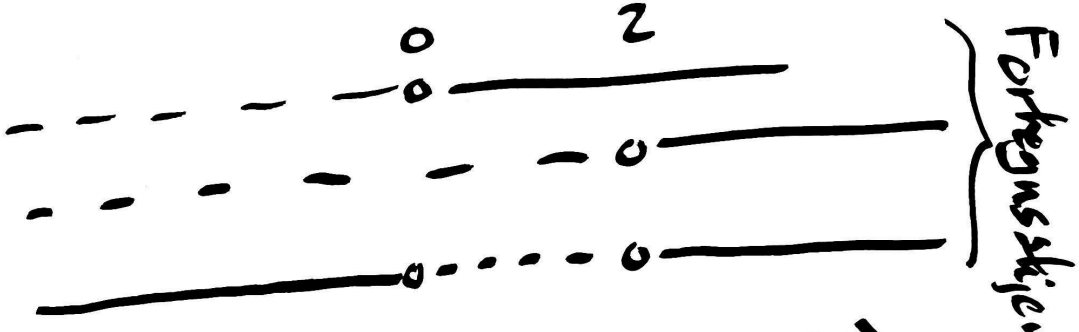
2.gradsulikhet.

$$x^2 > 2x$$

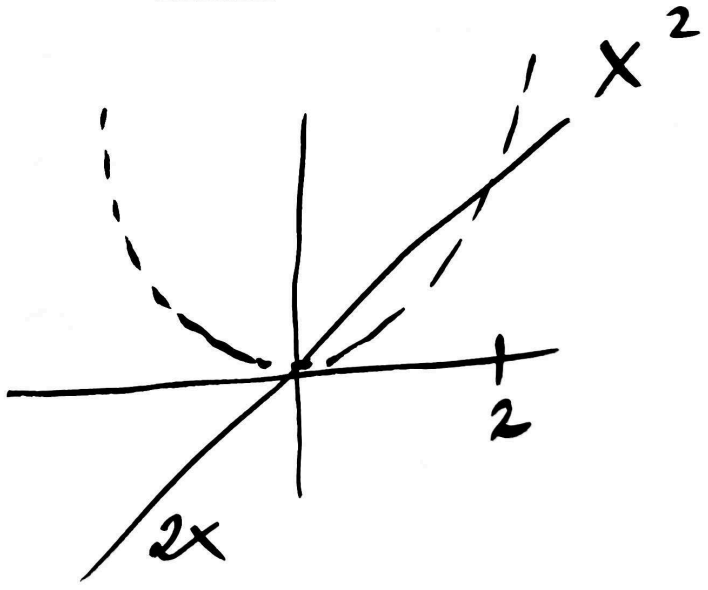
$$\Leftrightarrow \quad x^2 - 2x > 0$$

$$\Leftrightarrow \quad x(x-2) > 0$$

Fortegns-linjer



Løsningsmengden er $\langle -\infty, 0 \rangle \cup \langle 2, \infty \rangle$

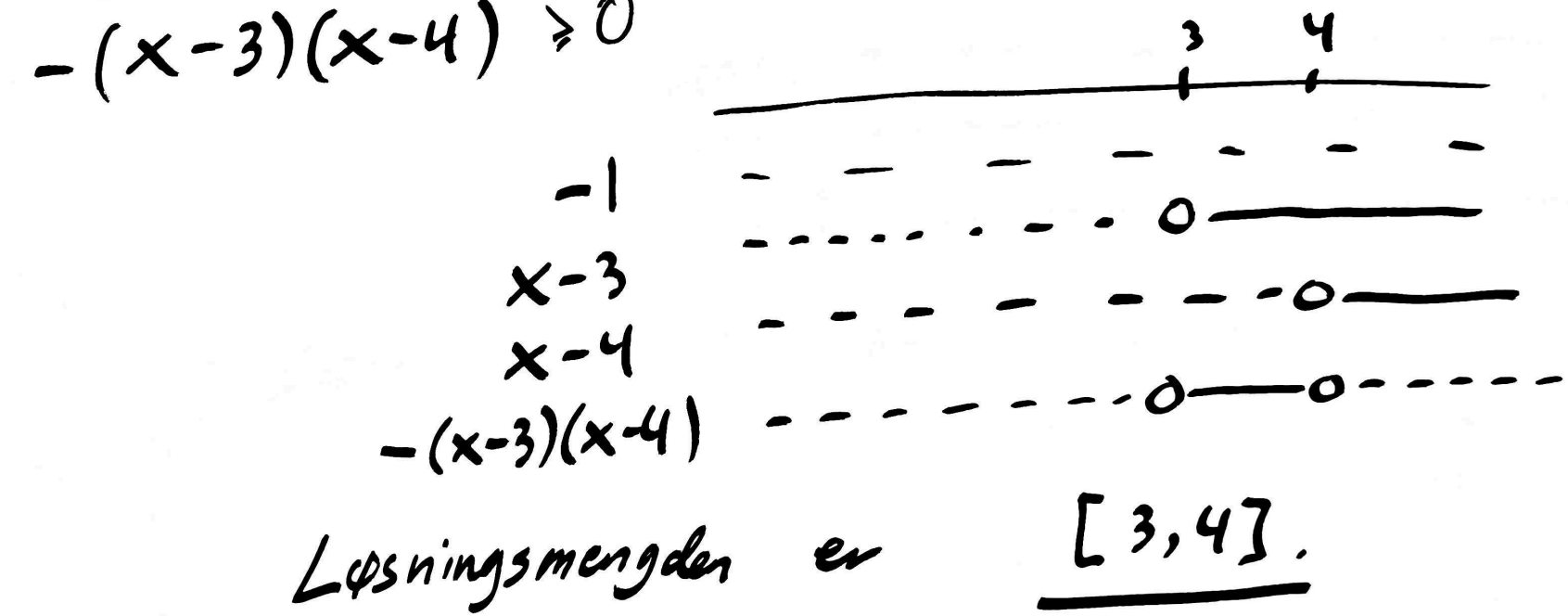


$$-x^2 + 7x \geq 12$$

$$-x^2 + 7x - 12 \geq 0$$

$$-(x^2 - 7x + 12) \geq 0$$

$$-(x-3)(x-4) \geq 0$$



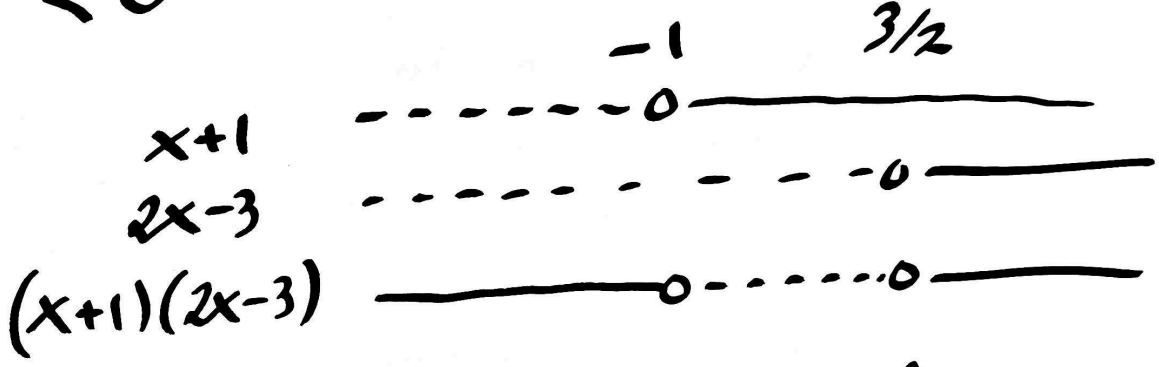
Løsningsmengden er $\underline{[3,4]}$.

$$3x^2 - 7x < x^2 - 6x + 3$$

$$3x^2 - x^2 - 7x + 6x - 3 < 0$$

$$2x^2 - x - 3 < 0$$

$$(x+1)(2x-3) < 0$$

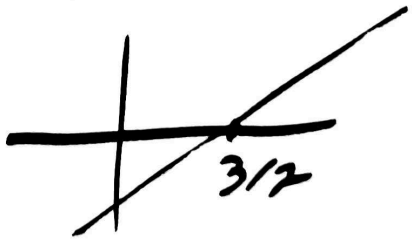


Lösningsmängden är $\underline{\langle -1, 3/2 \rangle}$

(Alternativt: $-1 < x < 3/2$)

(Förtegn till $2x-3$:

$2x - 3 = 0 \iff 2x = 3 \iff x = 3/2$

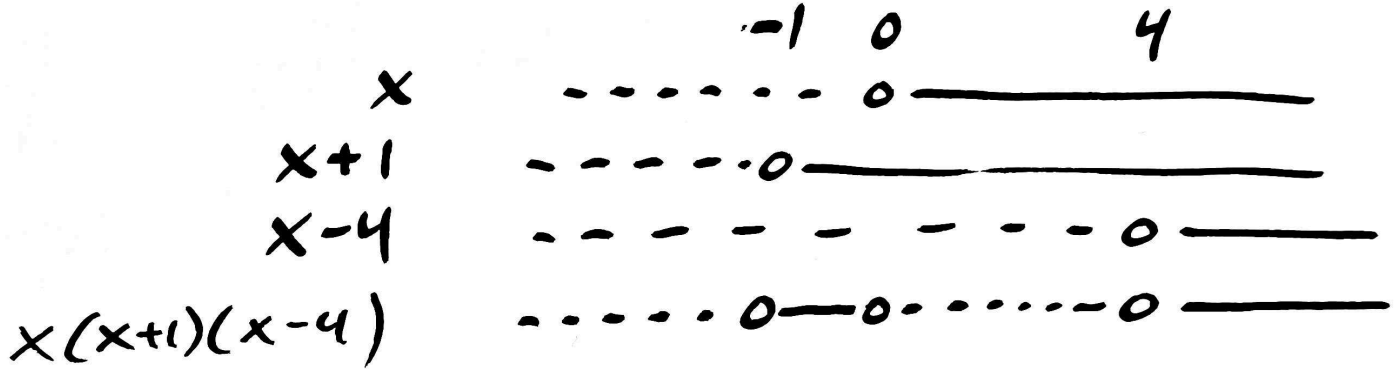


positivt stigningstall (2))

$$x^3 - 3x^2 - 4x > 0$$

$$x(x^2 - 3x - 4) > 0$$

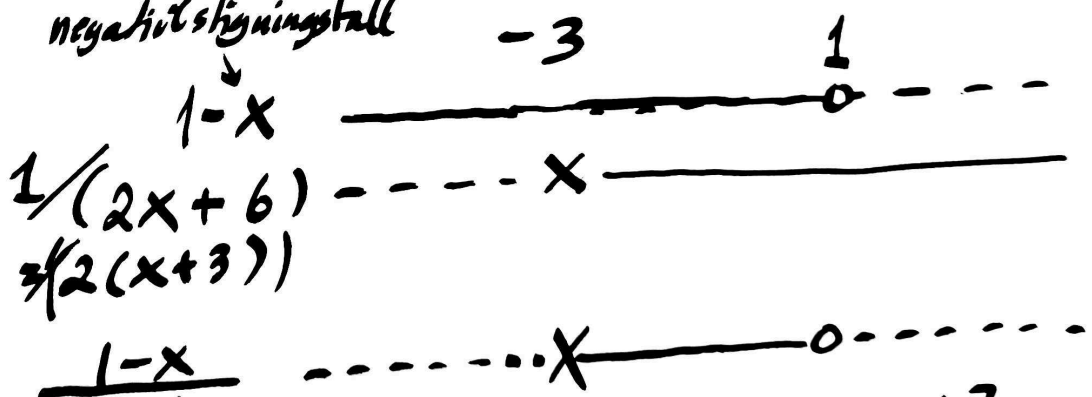
$$x(x-4)(x+1) > 0$$



Løsningsmengden er $\langle -1, 0\rangle \cup \langle 4, \infty\rangle$

$$\frac{1-x}{2x+6} \geq 0$$

negativt stigningstall



Løsningsmengden $\langle -3, 1]$

Eksamen
mai 25

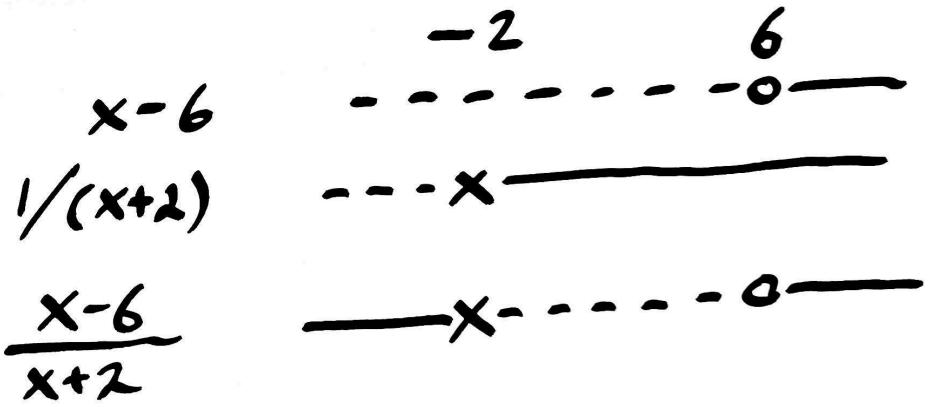
$$\frac{2x-4}{x+2} \geq 1 \qquad x \neq -2$$

$$\frac{2x-4}{x+2} - 1 \geq 0$$

$$\frac{2x-4}{x+2} - \frac{x+2}{x+2} \geq 0$$

$$\frac{(2x-4)-(x+2)}{x+2} \geq 0$$

$$\frac{x-6}{x+2} \geqslant 0$$

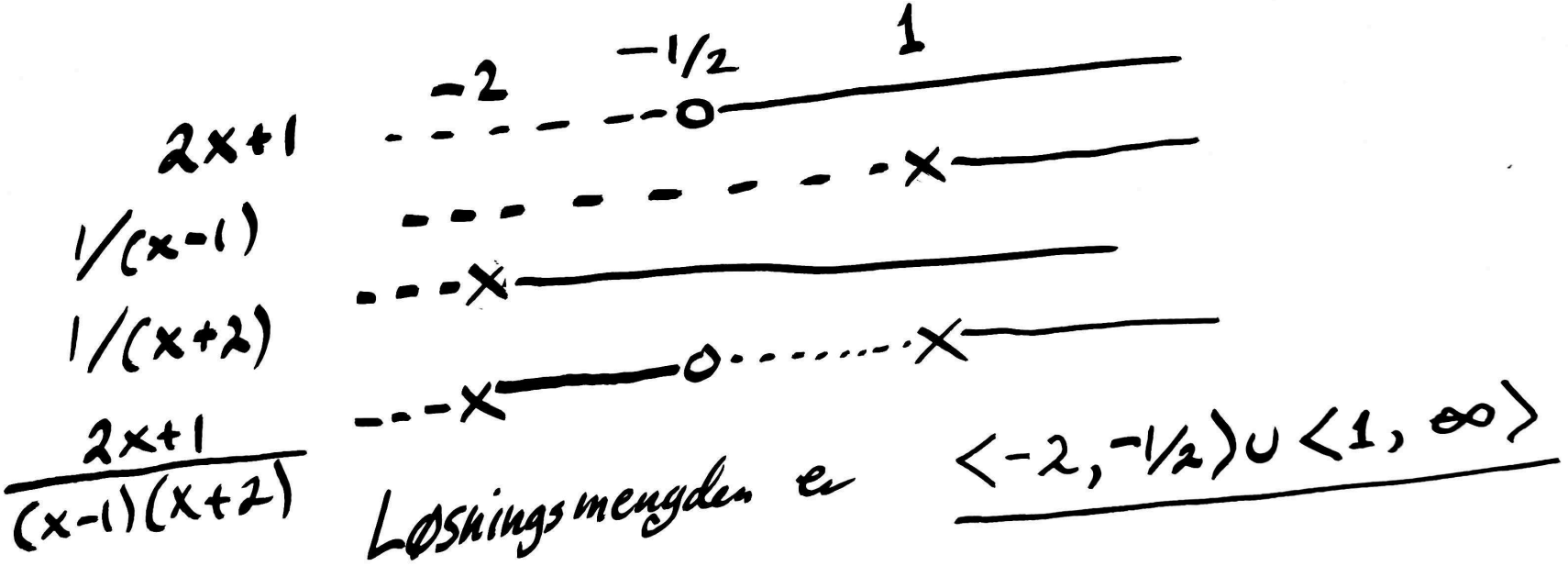


Løsningsmengden er $\langle -\infty, -2 \rangle \cup [6, \infty\rangle$

$$\frac{1}{x-1} + \frac{1}{x+2} > 0$$

$$\frac{x+2}{(x-1)(x+2)} + \frac{1\cdot(x-1)}{(x+2)(x-1)} > 0$$

$$\frac{x+2+x-1}{(x-1)(x+2)} > 0 \iff \frac{2x+1}{(x-1)(x+2)} > 0$$



Løsningsmengden er $\langle -2, -1/2 \rangle \cup \langle 1, \infty \rangle$