FORMELSAMLING FOR MATEMATIKK 1000

Regneregler for potenser

$$a^p\,a^q = a^{p+q}$$
$$a^p/a^q = a^{p-q}$$
$$a^{-q} = 1/a^q$$
$$(a^p)^q = a^{p\cdot q}$$
$$a^{1/p} = \sqrt[p]{a}$$
$$a^p\,b^p = (ab)^p$$
$$a^p/b^p = (a/b)^p$$

Regneregler for logaritmer

$$\ln(ab) = \ln a + \ln b$$
$$\ln(a/b) = \ln a - \ln b$$
$$\ln(a^p) = p\cdot \ln a$$
$$\log_a(b) = \ln(b)/\ln(a)$$

Eksakte verdier til sin og cos

u	u	$\sin u$	$\cos u$	$\tan u$
0	$0°$	0	1	0
$\pi/6$	$30°$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	$45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$60°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	$90°$	1	0	–

Trigonometriske formler

$$1 = \sin^2 u + \cos^2 u$$
$$\sin u = \sin(u + 2\pi n), \quad n \in \mathbb{Z}$$
$$\cos u = \cos(u + 2\pi n), \quad n \in \mathbb{Z}$$
$$\cos(u) = \cos(-u)$$
$$-\sin(u) = \sin(-u)$$
$$\sin(u) = \sin(\pi - u)$$
$$-\cos(u) = \cos(\pi - u)$$
$$\sin(u) = \cos(\pi/2 - u)$$
$$\sin(u+v) = \sin u\,\cos v + \cos u\,\sin v$$
$$\cos(u+v) = \cos u\,\cos v - \sin u\,\sin v$$
$$\sin(2u) = 2\sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$\cos^2 u = (1 + \cos(2u))/2$$
$$\sin^2 u = (1 - \cos(2u))/2$$
$$\tan u = \sin u/\cos u$$
$$\cot u = \cos u/\sin u$$
$$\csc u = 1/\sin u$$
$$\sec u = 1/\cos u$$

Likninger

Rett linje: $y - y_0 = a(x - x_0)$

Sirkel: $(x - x_0)^2 + (y - y_0)^2 = r^2$

Komplekse tall

$$z = a + ib = r(\cos\phi + i\sin\phi) = re^{i\phi}$$
$$r^2 = a^2 + b^2 \qquad \tan\phi = b/a \quad a \neq 0$$

Definisjon av den deriverte

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

Derivasjonsregler

$$(u+v)' = u' + v'$$
$$(c\,u(x))' = c\,u'(x) \quad (c \text{ konstant})$$
$$(u\cdot v)' = u'v + uv'$$
$$(u/v)' = \frac{u'v - uv'}{v^2}$$
$$(f(u))' = f'(u)\cdot u' \quad \left(\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx}\right)$$

Noen deriverte

$$(c)' = 0 \quad (c \text{ konstant})$$
$$(x^r)' = rx^{r-1}$$
$$(e^x)' = e^x$$
$$(a^x)' = a^x \ln a \quad (a > 0)$$
$$(\ln x)' = 1/x$$
$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(\tan x)' = 1/\cos^2 x = 1 + \tan^2 x$$
$$(\arcsin x)' = 1/\sqrt{1-x^2}$$
$$(\arctan x)' = \frac{1}{1+x^2}$$

Midpunktsformelen for derivasjon

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

L'Hôpitals regel

Hvis $f(x)$ og $g(x)$ begge går mot 0, eller de går mot ∞ eller mot $-\infty$ når $x \to a$, da er

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

forutsatt at grensen til høyre eksisterer.

Riemann-integral

$$\int_a^b f(x)\,dx = \lim_{|\mathcal{P}|\to 0} \mathcal{R}_{\mathcal{P},\mathcal{S}}$$

der $\mathcal{R}_{\mathcal{P},\mathcal{S}} = \sum_{i=0}^n f(x_i^*)\Delta x_i$

Egenskaper til bestemte integral

$$\begin{aligned}
\int_a^b F'(x)\,dx &= F(b) - F(a) \\
\int_a^b cf(x)\,dx &= c\int_a^b f(x)\,dx \\
\int_a^b (f+g)\,dx &= \int_a^b f\,dx + \int_a^b g\,dx \\
\int_a^b uv'\,dx &= [uv]_a^b - \int_a^b u'v\,dx \\
\int_a^b f(u)u'\,dx &= \int_{u(a)}^{u(b)} f(u)\,du
\end{aligned}$$

Noen antideriverte

$$\begin{aligned}
\int x^r\,dx &= \frac{x^{r+1}}{r+1} + C \quad r \neq -1 \\
\int \frac{1}{x}\,dx &= \ln|x| + C \\
\int e^x\,dx &= e^x + C \\
\int \sin x\,dx &= -\cos x + C \\
\int \cos x\,dx &= \sin x + C \\
\int \frac{1}{\cos^2 x}\,dx &= \tan x + C \\
\int \frac{1}{\sqrt{1-x^2}}\,dx &= \arcsin x + C \\
\int \frac{1}{1+x^2}\,dx &= \arctan x + C
\end{aligned}$$

Egenskaper til ubestemte integral

$$\begin{aligned}
\int uv'\,dx &= uv - \int u'v\,dx \\
\int f(u)u'\,dx &= \int f(u)\,du
\end{aligned}$$

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$$

Trapesmetoden

$$\int_a^b f(x)\,dx \approx T_n = \frac{b-a}{n} \cdot \left(\frac{f(x_0)}{2} + f(x_1) + \cdots + f(x_{n-1}) + \frac{f(x_n)}{2}\right)$$

Lengde, Areal og Volum til objekter bestemt av $f(x)$

$$\begin{aligned}
L &= \int_a^b \sqrt{1+(f'(x))^2}\,dx \text{ (buelengde)} \\
A &= 2\pi \int_a^b |f(x)|\sqrt{1+(f'(x))^2}\,dx \\
V &= \pi \int_a^b f(x)^2\,\mathrm{d}x \text{ (rotasjon om } x\text{-aksen)} \\
V &= 2\pi \int_a^b x|f(x)|\,\mathrm{d}x \text{ (rotasjon om } y\text{-aksen)}
\end{aligned}$$

Gjennomsnitt: $\overline{y} = \frac{1}{b-a}\int_a^b y(x)\,dx$

Newtons metode

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Matriser For en 2×2-matrise:

$$\begin{aligned}
\begin{vmatrix} a & b \\ c & d \end{vmatrix} &= ad - bc \\
\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} &= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\end{aligned}$$

Differensiallikninger

$h(y) \cdot y' = g(x)$ gir $\int h(y)\,dy = \int g(x)\,dx$

$y' + f(x)y = g(x)$ gir $\left(ye^{F(x)}\right)' = g(x)e^{F(x)}$

Løsninger til likningen $y'' + py' + qy = 0$:

$$\begin{aligned}
y &= Ae^{r_1 x} + Be^{r_2 x} \quad \text{(to reelle røtter)} \\
y &= e^{rx}(A + Bx) \quad \text{(ei rot)} \\
y &= e^{ax}(A\cos(bx) + B\sin(bx))
\end{aligned}$$

(to komplekse røtter $r = a \pm ib$)

Eulers metode

$y_{n+1} = y_n + F(x_n, y_n)h$ der

$y'(x) = F(x,y)$ og $x_n = x_0 + nh$

Taylor-polynomer

$$\begin{aligned}
f(x) &= P_n(x) + R_n(x) \\
P_n(x) &= \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k \\
R_n(x) &= \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}
\end{aligned}$$

Lineær tilnærming

$$f(x) \approx f(a) + f'(a)(x-a)$$